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THE DESIGN OF TALL STEEL BUILDING FRAMES FOR STABILITY

BY

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled "DESIGN OF TALL STEEL BUILDING FRAMES FOR STABILITY" submitted by BRIAN ROSS WOOD in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

Dedicated to my wife, Muriel, and my children,

Wende, Heather, Glen, and Douglas.

ABSTRACT

This investigation attempts to evaluate procedures related to the design of tall steel building frames for stability. Behavior of frames under various loading conditions are discussed along with analyses used to predict this behavior.

Traditional procedures used for the design of steel structures for stability are outlined along with proposed modifications of these design procedures. A model that can be used to predict the ultimate strength of restrained columns permitted to sway is outlined. This model is used to evaluate the predictions of various design methods.

The $P\Delta$ procedure for the design of steel structures for stability is outlined. The rational and accuracy of this method is investigated. A model for the analysis of inelastic sway subassemblages is developed and the effect of inelastic beam response on the behavior of restrained sway members is studied. A stability check for use with the $P\Delta$ procedure is proposed.

Examples of a multi-storey building frame designed using different design methods are presented along with a second order elastic-plastic analysis of these frames. Recommendations for the design of tall steel structures for stability are made.

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LIST OF SYMBOLS

A	cross sectional area of member
C	stability function
C_e	Euler buckling load
C_f	factored axial load
C_r	factored compressive resistance of the member
d	depth of the section
E	modulus of elasticity
f	shape factor
G, G_u, G_L	coefficients required for effective length calculation
H'_i	fictitious sway force at level i
H_n	lateral load applied at storey n
h	storey height
I	moment of inertia of member
I_c	column moment of inertia
I_G, I_B	girder moment of inertia
K	effective length factor
K_F	effective length factor calculated on the basis of frame buckling
K_S	effective length factor calculated on etu basis of storey buckling
K_T	effective length factor calculated using traditional methods
$[K]$	stability stiffness matrix

L	number length
L_c	length of column
L_G, L_B	length of girder
M_{AB}, M_{AD}	bending moment
M_f, M_{fx}, M_{fy}	factored bending moment
M_{f1}	smaller factored end moment of a beam column
M_{f2}	larger factored end moment of a beam column
M_n	moment at the column top in storey n
M_p	plastic moment capacity of member
M_{pc}	reduced plastic moment in the presence of axial load
M_r, M_{rx}, M_{ry}	factored bending resistance; joint resisting function
M_T	moment at column top
P, P_n	column axial load
P_{cr}	critical axial load
Pe'	Euler buckling load
ΣP_i	sum of the axial loads at level i
P_{pL}	rigid plastic collapse load
$\{P\}$	primary axial forces
P_y	axial yield load for the member
Q_{AB}	uniformly distributed load applied to member AB
Q_n	lateral shear carried by the member in storey n
r, r_x	radius of gyration

R1	sway effects ratio
S	stability function
X	distance from support to the segment centre
$\{X\}$	unrestrained joint deformations
V_{AB}	end shear for member AB
V_i'	artificial shear at level i
V_i	deflection in segment i
w_p	uniform load required to form a plastic hinge
Z	plastic section modulus
α	load factor
α_i	slope of segment i
β	moment amplifier
γ	column end rotation
Δ, Δ_i	sway deflection
δ_i	length of segment i
θ	joint rotation
λ	load factor
σ_y	yield stress
ϕ	curvature
$\omega, \omega_x, \omega_y$	equivalent moment factor

CHAPTER I

INTRODUCTION

With the construction of many taller buildings, an increased concern has become evident about the overall stability of these increasingly slender structures. Confusion in this aspect of the design of tall buildings has arisen from the fact that the analyses used by the designer to compute the distribution of forces and moments throughout the structure do not include those aspects of the behavior of the structure relating to the overall stability of the structure. Thus, the results of these analyses must be adjusted empirically to reflect the stability effects (1).

Traditionally, the adjustment for the neglect of stability effects is made in the interaction equations for the design of beam columns (24). First, the structure must be classified as either "sway permitted" or "sway prevented". Then, the effective length factor, K, and the equivalent moment factor, ω, are calculated depending on how the structure is classified. These factors are then used in the stability interaction equation (24,43):

$$\frac{C_f}{C_r} + \frac{\omega M_f}{M_r\left(1 - \frac{C_f}{C_e}\right)} \leq 1.0 \qquad (1.1)$$

where:

M_f = maximum end moment on the member produced by the factored loads

C_f = axial load on the member produced by the factored loads

C_r = the factored compressive resistance developed by the member if

subjected to an axial load

M_r = the factored moment resistance developed by the member if subjected to loads causing bending moments without significant axial force

C_e = $\frac{286{,}000A}{(KL/r_x)^2}$, the elastic buckling strength of the member where KL/r_x represents the slenderness ratio in the plane of bending.

Traditionally, it had been assumed that if the structure contained a stiff vertical truss or shear wall, translation would be effectively prevented and the columns could then be designed as "sway prevented" members. In the absence of a stiff vertical element, resistance to lateral sway must be provided by the flexural resistance of the beams and columns in the structure and the columns were then designed as "sway permitted" members.

The use of various terms in Equation (1.1) to adjust for the neglect of the stability effects in the analyses of the structure has been an area of considerable controversy, particularly, when considering the design of beam columns in tall structures (3,4). As a result, there have been a number of proposals to rationalize and improve the design of beam columns for stability effects (5,6,7,8,9). Most of the attention given to the problem has focused on attempts to correlate the mathematical bifurcation of equilibrium states with the actual stability problem. The main object of many of these proposals has been to develop a more rational effective length or K factor to use in the interaction equations. The use of the K factor in the interaction equations has been based on empirical considerations, and thus, any change in the K factor itself must reflect the nature of its use.

Recently, a departure from the traditional approach has been proposed (10,11,12,38,39). This method has been called the PΔ method for stability design.

The method proposed that either a rigorous or an approximate second order analysis be performed on the structure so that the effect of axial loads on the deformed geometry of the structure is considered. Thus, the distribution of moments and forces in the members of a structure will include those most significant stability effects. It is further proposed that if the stability effects have been included in the analyses, the members may be designed on the basis of the "sway prevented" condition while if these effects have not been included, the structure must be designed as "sway permitted". It is also suggested, therefore, that the classification of structures as "sway permitted" or "sway prevented" should be based only on whether stability effects are included in the analysis of the structure.

The purpose of this dissertation is to study the implications of the proposed PΔ method and to propose any modifications or limitations that may be required. Included in this study will be an evaluation of other design methods, both existing and proposed.

Before exploring the use of the PΔ method, an understanding of the behavior of structures under various loading systems is required. An outline of the behavior of frames subjected to both vertical loads and to a combination of horizontal and vertical loads is presented in Chapter II. This behavior is illustrated by the results of a number of experiments performed on various types of frames (16,17,18,20).

The traditional method of accounting for stability effects is

outlined in Chapter III. In this Chapter, a model is developed for predicting the ultimate strength of restrained beam-column permitted to sway. An elastic-plastic analysis of this model is used to illustrate the manner in which the traditional methods account for second order effects. This model is also used throughout the dissertation to illustrate various factors effecting the strength and behavior of beam-columns.

As a result of the extensive research into the behavior of beam columns, many proposals have been made to the design of such members for stability effects. As suggested by Lay (3), when a new method for design is introduced, the pros and cons of this method are seldom discussed with respect to those of other proposals. The object of Chapter IV is to discuss various recent proposals and to relate the implications of these proposals to those of the PΔ technique.

The PΔ method is outlined in detail in Chapter V as originally proposed. The investigation of various limitations of the PΔ method is described in Chapter V. The results of analyses are introduced to confirm the method, and the accuracy and convergence of the approximate second order analysis is discussed. The significance of the various second order effects considered are reviewed. The effect of inelastic beam behavior on column restraint, the effects of initial out of plumbs, the effect of axial loads on column stiffness, the effect of overall frame stiffness, and other factors that may have some influence on stability of frames are discussed.

The lack of specific information on the effect of inelastic beam response on the behavior of beam-column subassemblages lead to the development of an analysis in Chapter VI to predict such behavior. The

results of the analysis are confirmed by comparison to various experimental results. The results of the analyses are then used to examine the effect of inelastic beam response on the stability of beam-column subassemblages.

To illustrate the use of the P-Delta technique for the design of tall building frames, design examples are presented in Chapter VII. In this chapter, a 26-storey building frame is designed using both the P-Delta technique and traditional design techniques and a proposal is presented for designing without consideration of second order effects. The resulting member sizes are compared and the final designs are analyzed using a second order elastic-plastic analysis to compare the behavior of the different frames. In addition, a "leaned frame" is also designed using the P-Delta technique to examine the behavior of a frame subjected to large sway effects.

The recommendations developed from the investigation are presented in Chapter VIII. A summary of the investigation and conclusions reached are presented in Chapter IX.

CHAPTER II

BEHAVIOR OF FRAMES

2.1 INTRODUCTION

The significance of second order effects is more clearly understood when related to the behavior of frames under various loading conditions. Of particular importance is the relationship between the behavior of frames as predicted by various analytical models and the actual behavior of frames as observed in experimental models.

Extensive research has been conducted in recent years to investigate the behavior of unclad multi-storey frames and their components (13,27). This research has included both analytical and experimental studies. The experimental approach has been to test relatively large-scale simple frames, designed so that the stability effects are similar to those predicted in portions of multi-storey frames.

In this chapter, a number of analytical models used to predict the behavior of frames will be discussed. The behavior predicted by these analyses shall then be compared with experimental behavior to demonstrate the significance of second order effects to the response of building frames.

2.2 ANALYSIS OF FRAMES

The behavior of a frame can be described in terms of the relationship between applied loads and the resulting deformations (1). The prediction of this relationship depends on the material properties, loading history, and considerations of equilibrium and compatibility.

The discrepancy between the behavior predicted by the analysis and the "true" behavior as observed during tests depends on the sophistication of the mathematical model used in the analysis, (31). The sophistication of the model used is dependent on the desired accuracy and the availability of computational facilities and funds, (31). For design, the model used should include all those factors which significantly influence the behavior of the actual structure.

To illustrate the behavior of a frame as predicted by various mathematical models, the simple frame shown as an inset to Figure 2.1 will be used. The behavior of this frame can be described by the relationship between increasing load, P, and the lateral movement of the frame at the top, Δ.

Of particular importance to the analysis are assumptions involving material properties and frame geometry. When material properties are said to be elastic the relationship between stress and strain for the material shall be considered to be linear without limit. When the material is assumed to be elastic-plastic, the stress strain relationship will be linear until the plastic limit, then followed by a region of unlimited deformation at constant stress.

When a first order analysis is used, the loads are considered to act on the undeformed geometry of the frame. In the case of the second order analysis, the effect of loads shall be considered to act on the deformed or deflected shape of the frame.

If the frame in Figure 2.1 is analyzed for increasing vertical loads and γ set to zero, members initially geometrically perfect, using a first order elastic analysis, the solution of this frame would

give zero displacements for all loads except the critical loads, (31). At the critical loads, the deflections of the frame are indeterminate and the load-deformation curve bifurcates or divides into two branches, (31). Real frames do not bifurcate in such a manner as they are never geometrically perfect and this bifurcation only results from a mathematical representation of the real frame, (31). The elastic buckling load is, however, a useful tool in predicting a upper limit of member behavior. The elastic buckling load, P_{cr}, is shown as the upper dashed line in Figure 2.1.

If the lateral load is increased in proportion with the vertical loads, and a first order elastic analysis is used, the load deformation relationship shall be linear as shown in Figure 2.1 as curve A. When a second order elastic analysis is used, however, the load deformation response of the frame will be non-linear as shown in Figure 2.1 as curve B. In the limit approaching infinite deformation, the second order elastic response approaches the elastic buckling load, P_{cr} (1).

If the material response of the frame is no longer perfectly elastic, another critical load may be reached before the elastic buckling load. This critical load results from the formation of a plastic mechanism in the frame (1). The critical load is shown as P_{pl} in Figure 2.1 for a rigid plastic analysis. As in the case of the buckling load, the deflections are indeterminate for a rigid plastic first order analysis. If a second order rigid plastic analysis is used, the loads must decrease to maintain equilibrium as shown in Figure 2.1 (1).

As a result, when a first order elastic-plastic analysis is used, the load deformation response predicted for the frame is shown as curve C. The limit of the first order elastic-plastic analysis is the first order rigid plastic limit. The results of a second order elastic-plastic analysis are shown as curve D. The difference between the first order and second order elastic-plastic curves is a measure of the PΔ effects for a given frame and loading condition (1).

The "true" frame behavior is depicted by curve E as shown in Figure 2.1. The discrepancy between the results of a second order elastic-plastic analysis and the "true" behavior is the result of gradual penetration of the yielded zones, residual stresses, initial imperfections and strain hardening. This difference will vary depending on particular frame geometry and stiffness, material properties, and the loading conditions.

2.3 BEHAVIOR OF FRAMES SUBJECTED TO COMBINED GRAVITY AND LATERAL LOADS

2.3.1 UNBRACED FRAMES

To examine the behavior of an unbraced frame subjected to combined loading, a simple single-storey frame will be used (16). The test of this frame has been designed so that the stability effects are similar to those predicted in portions of multi-storey frames.

The inset to Figure 2.2 shows a single-storey, single-bay frame subjected to constant vertical loads and to a lateral load, H, which deflects the frame in a sidesway mode (16). The lateral deflection at the beam level is denoted by Δ. The upper broken lines depict the behavior predicted by a first order elastic-plastic analysis, while,

lower dashed curve represents that predicted by a similar second order analysis. The results observed during the test of the frame are represented by the full lines joining the solid circles.

The significant stages in the response of the frame are denoted by A, B, and C in Figure 2.2. Stage A on the actual response curve is the stage corresponding to the (nominal) formation of the first plastic hinge. The corresponding stage, as predicted by the first order analysis, is denoted as B in Figure 2.2. As shown, the inclusion of second order effects has significantly reduced the load corresponding to the attainment of the ultimate capacity in the critical member. These effects have also produced increases in the deflections and corresponding moments and shears throughout the frame at each stage of loading.

At Stage C, the structure has reached its maximum capacity. Beyond this stage, equilibrium can only be maintained by a reduction of the lateral load. This stage is assumed to correspond to failure of the structure, although under particular loading conditions, this is not necessarily the case (1).

The structure shown in the inset to Figure 2.2 was designed plastically so that full moment redistribution could occur beyond Stage A. In a similar structure designed according to elastic design techniques, Stage A would correspond to the limit state of the frame since the lateral bracing spacing and plate slenderness provisions are sufficient only for the critical member to reach its ultimate capacity and permit only minor redistribution of moments.

2.3.2. FRAMES WITH STIFF VERTICAL BRACING ELEMENTS

A number of tests have been performed on frames which contain vertical bracing systems (17, 18). An example is the frame shown as an inset to Figure 2.3 (18) which contained a concrete shear wall to provide resistance to lateral movement. The test was designed to simulate a portion of a multi-story building. The vertical loads were held constant, while the horizontal loads were applied to the frame in increments. The experimental response is shown as the solid line joining the circles. As in the previous example, Stage A represents the formation of the first plastic hinge in the test, while Stage B represents the prediction of the first hinge by the first order elastic analysis. Stage C is the ultimate load of the frame.

The results of the first order elastic-plastic analysis are shown by the upper broken line. The results of a second order elastic-plastic analysis are shown by the dashed line. The second order analysis gives an adequate prediction of the behavior of the frame. Thus, inspite of the presence of a stiff vertical shear wall, the second order effects cause a significant difference between the first order prediction and test result.

Similar results were observed in tests of braced steel frames (17). In these studies, simple steel frames with diagonal bracing were subjected to increasing horizontal loads, one frame with columns subjected to axial loads, another frame with no axial loads on the columns. The load deformation behavior of these frames was significantly different. The differences in behavior were mainly due to the second order effects introduced by the presence of the axial loads (17).

A theoretical study was performed by Davison and Adams to examine the stability of braced multi-storey frames (14). For this study, an elastic-plastic second order analysis was developed to predict the behavior of both braced and unbraced frames. This analysis was used to predict the behavior of two twenty-four storey building frames, one braced and one unbraced, each designed to resist the same loading conditions (13). The results of these analyses showed that the diagonally braced frame deflected more than the unbraced frame at all loads up to the factored design loads. As a result, a comparison of first and second order analyses of the braced twenty-four storey frame showed that the ultimate load factor was reduced 36% by the $P\Delta$ effects.

2.4 BEHAVIOR OF FRAMES SUBJECTED TO GRAVITY LOADS ONLY

As outlined in section 2.2, when a geometrically perfect frame is subjected to concentric axial loads such as the frame shown as an inset to Figure 2.4, the deflections are zero until a critical load is reached, P_{cr}. At the critical load, however, any minor perturbation will cause the frame to lurch into a sidesway deflected shape (19). If small deflection theory is used to determine the critical load, the value of the sidesway deflection is indeterminate as implied by the horizontal dashed line in Figure 2.4. The large deflection theory, however, would predict the load deflection curve shown by the solid line in Figure 2.4. This implies a slight increase in load carrying capacity after the critical load is attained, but as the frame deflects further, inelastic action decreases frame stiffness and a decrease in the equilibrium load results (19).

Real frames, however, are not perfect. Initial curvatures are introduced in individual members during manufacture. Initial sways are introduced to members during erection, and loads are seldomly applied in a concentric manner as assumed in the analysis. As a result, the imperfect frame begins to sway laterally with increasing applied loads. At low loads, the sway is usually small, however, as inelastic action occurs in various members of the frame with a corresponding reduction of frame stiffness, the deflections increase rapidly as shown by the broken curve in Figure 2.4. As the load approaches the critical value, the deflections increase rapidly and the frame fails through inelastic stability (19).

Large-scale tests of a three storey, unbraced frame subjected to vertical loads only (shown as an inset to Figure 2.5) were performed by McNamee (20). As both frame geometry and applied loads are symmetrical, a first order analysis will not predict deflections. The ultimate strength can be predicted by rigid plastic analysis. The ultimate strength would be reached when a beam mechanism forms at P = 27 kips.

The load deflection relationship observed from test results, however, shows the frame deflecting with increasing load as a result of initial imperfections introduced during fabrication and erection. A second order analysis can be performed by assuming small lateral loads (¼ of 1% of P) are applied to the frame to simulate initial eccentricities (19). The results of this second order analysis are shown as the dashed line in Figure 2.5.

During the test, the first plastic hinge occurred at Stage

A, and failure occurred at Stage B as a result of inelastic instability. The behavior of the frame was quite similar to the behavior of a frame subjected to combined loading and the second order elastic-plastic analysis gave a realistic prediction of this behavior.

2.5 SUMMARY

As outlined in the previous section of this chapter, a first order prediction of the behavior of tall building frames overestimates both stiffness and strength of the frame, even when the frame contains a stiff vertical bracing element. In each case, the inclusion of the P-Delta effects in the analysis has resulted in a more realistic prediction of frame behavior. In addition, it has been shown that behavior of frames subjected to gravity loads is similar in all respects to that of frames with combined loading.

When the moments and forces used for member design are obtained from the results of a first order analysis, the design procedures must include an allowance for second order effects. Traditional procedures used to account for the neglect of second order effects will be outlined in Chapter III. Proposals to modify the traditional procedures will be outlined and examined in Chapter IV. A proposal to include the second order effects directly in analysis of structures will be outlined in Chapter V.

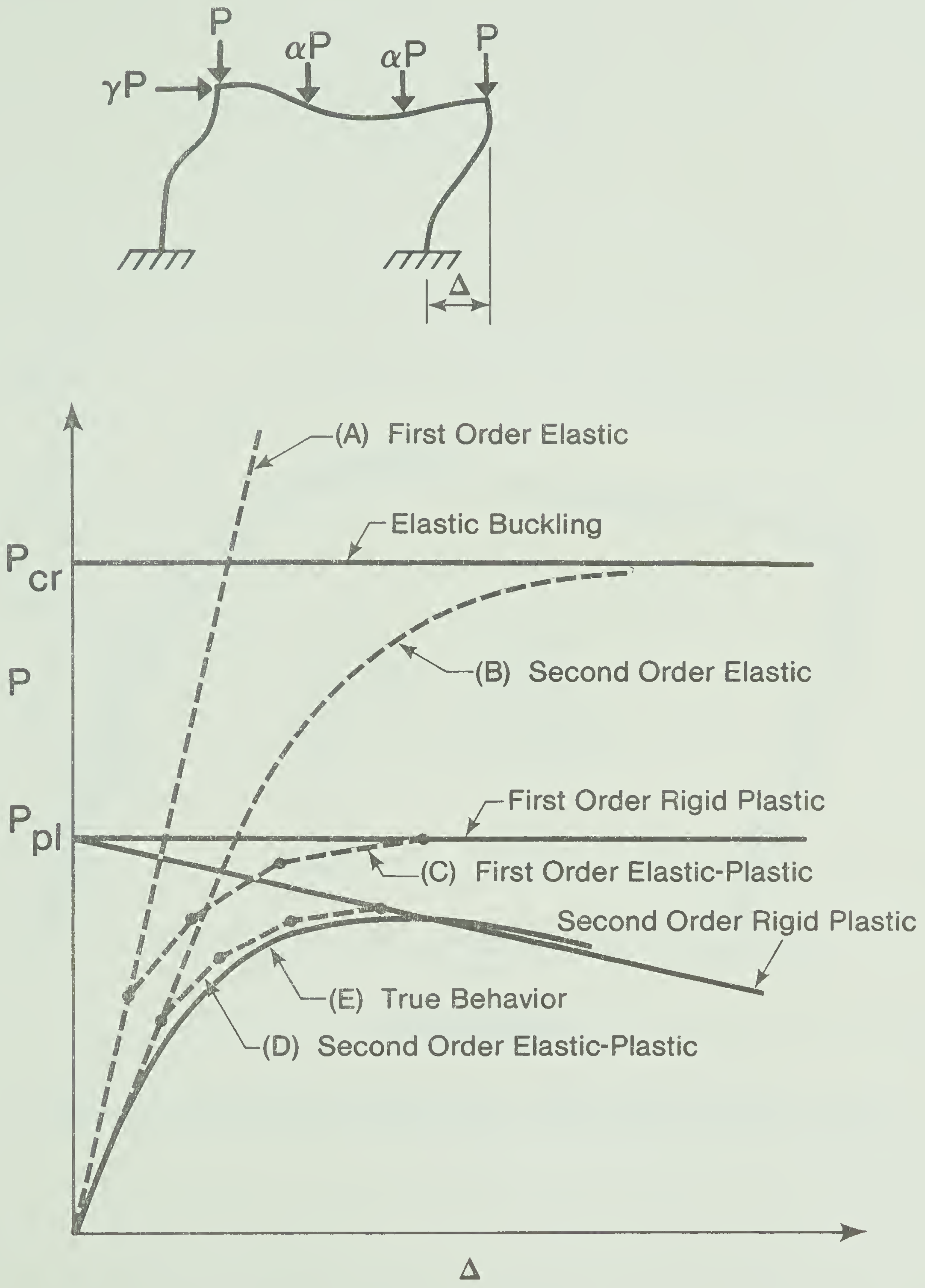


Figure 2.1 Load Deflection Relationships of Frames

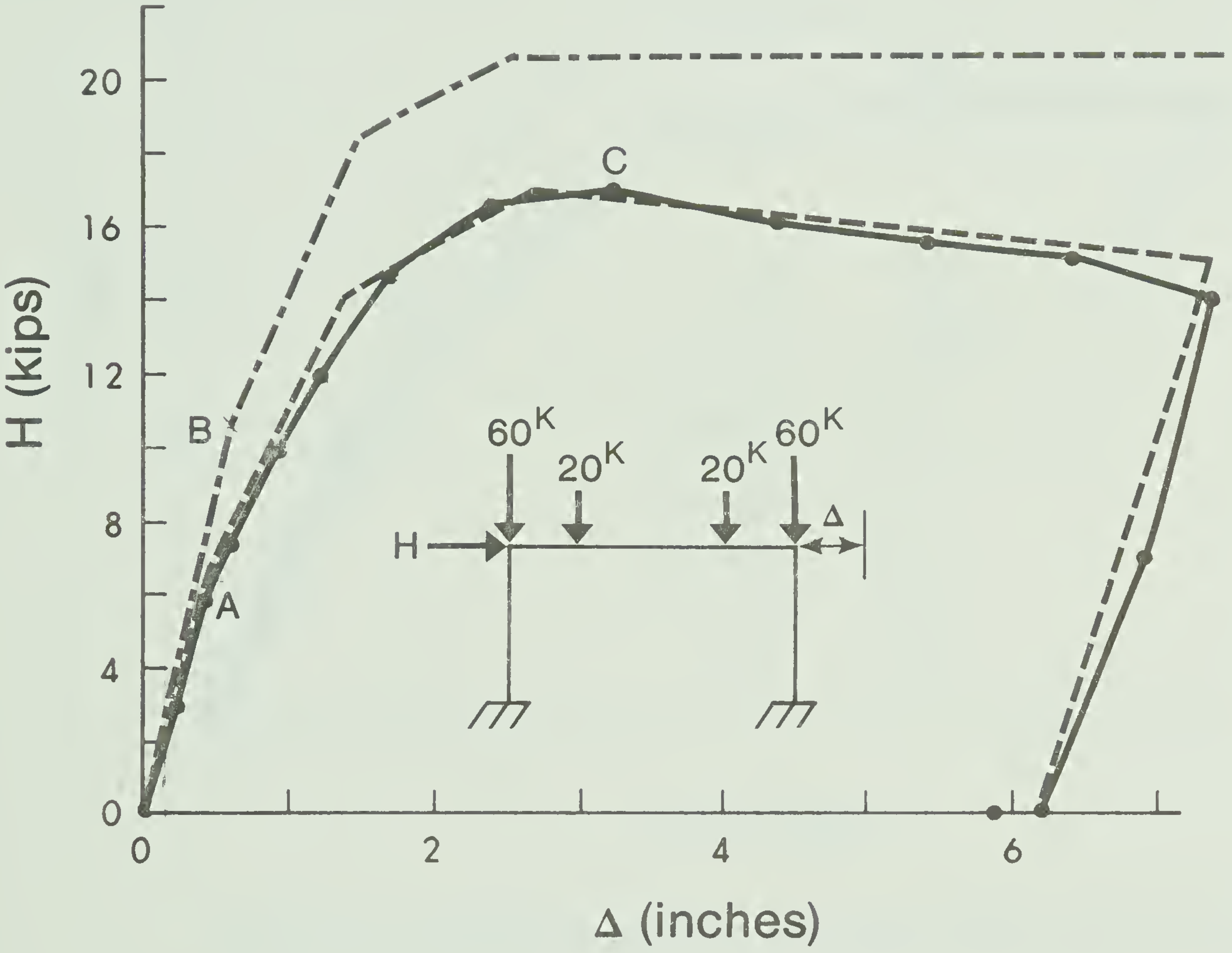


Figure 2.2 Load-Deflection Relationships - Rigid Frame, Combined Loading

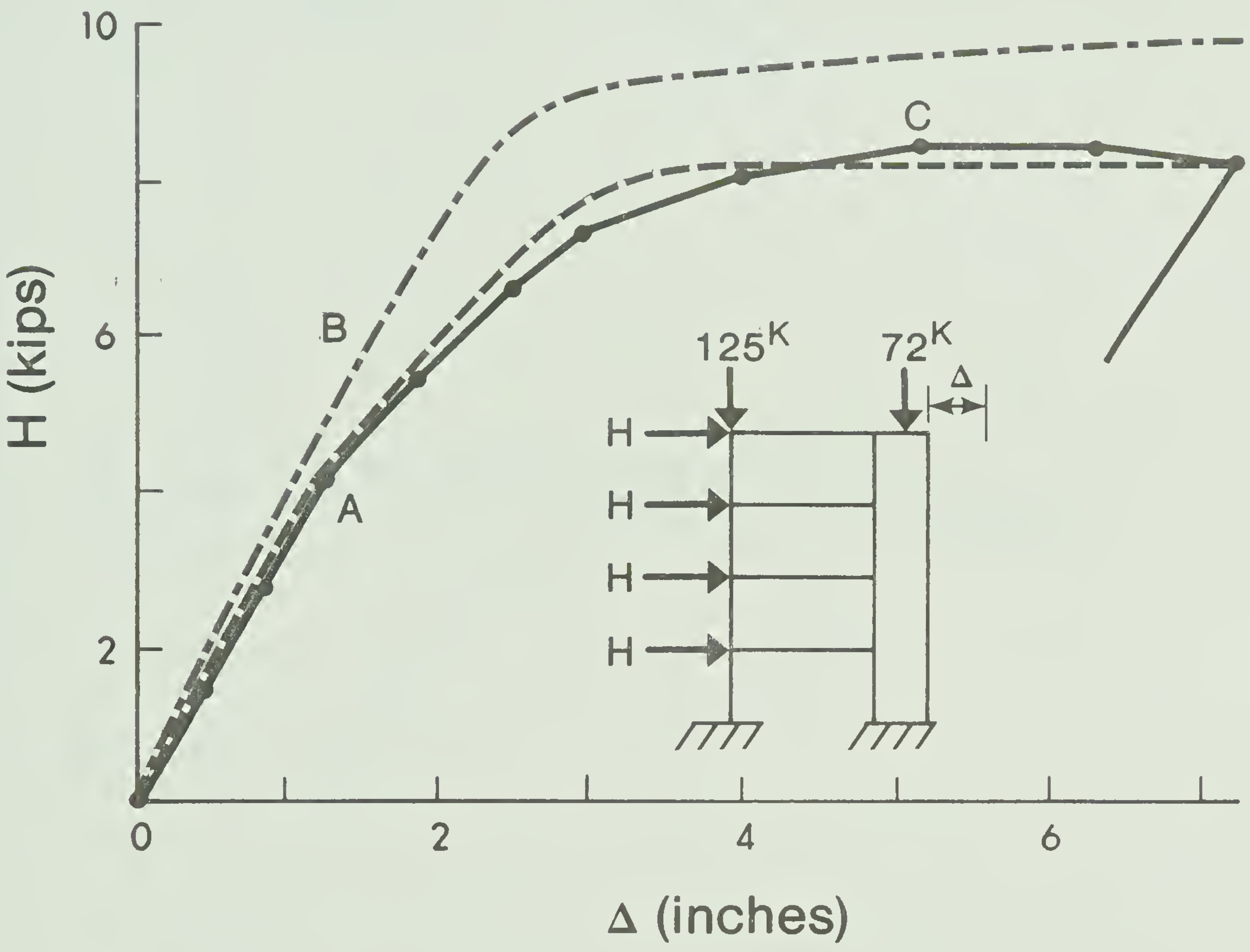


Figure 2.3 Load-Deflection Relationships Braced Frame, Combined Loading

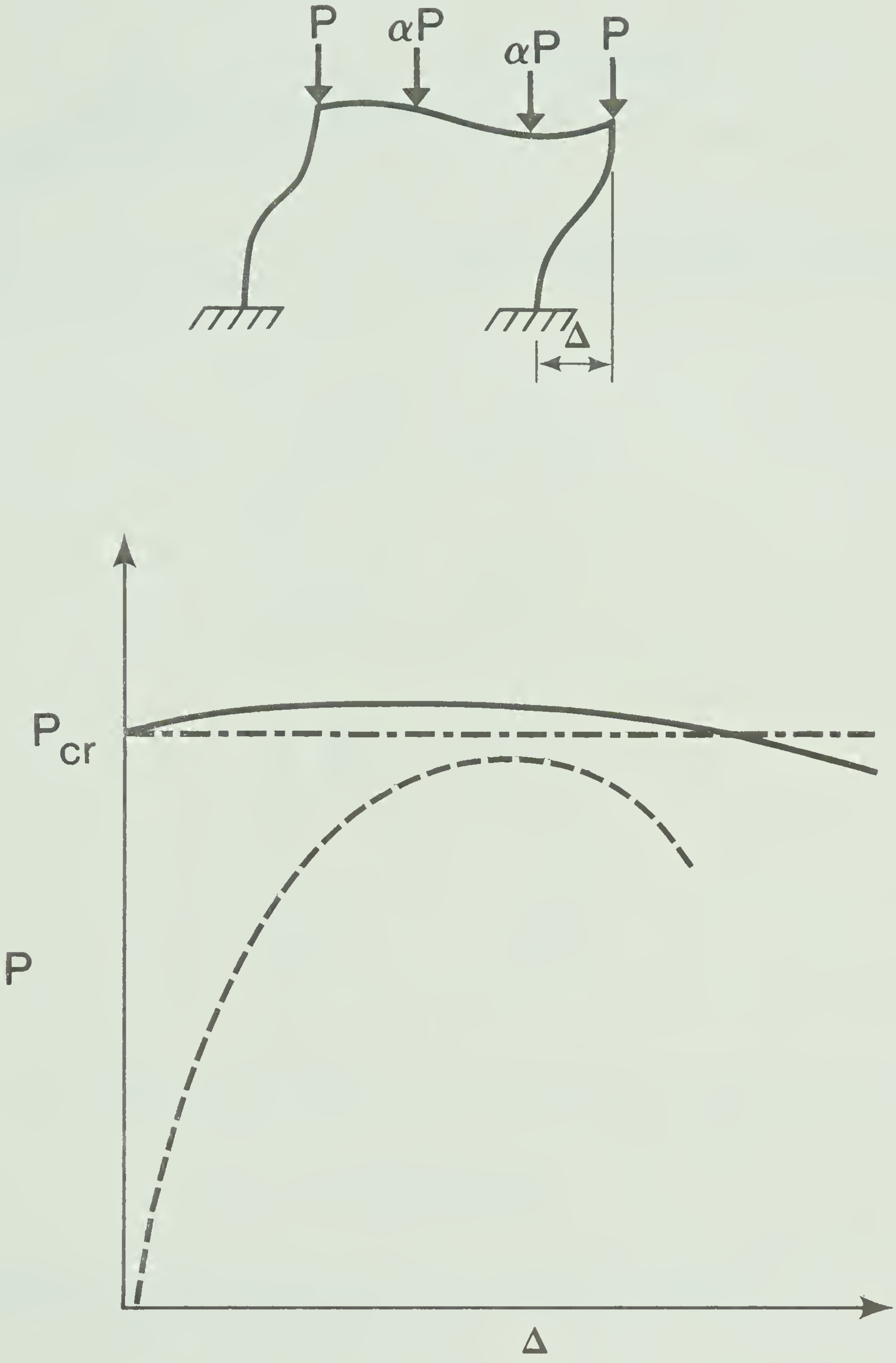


Figure 2.4 Load-Displacement Relationship, Rigid Frame, Gravity Load Only

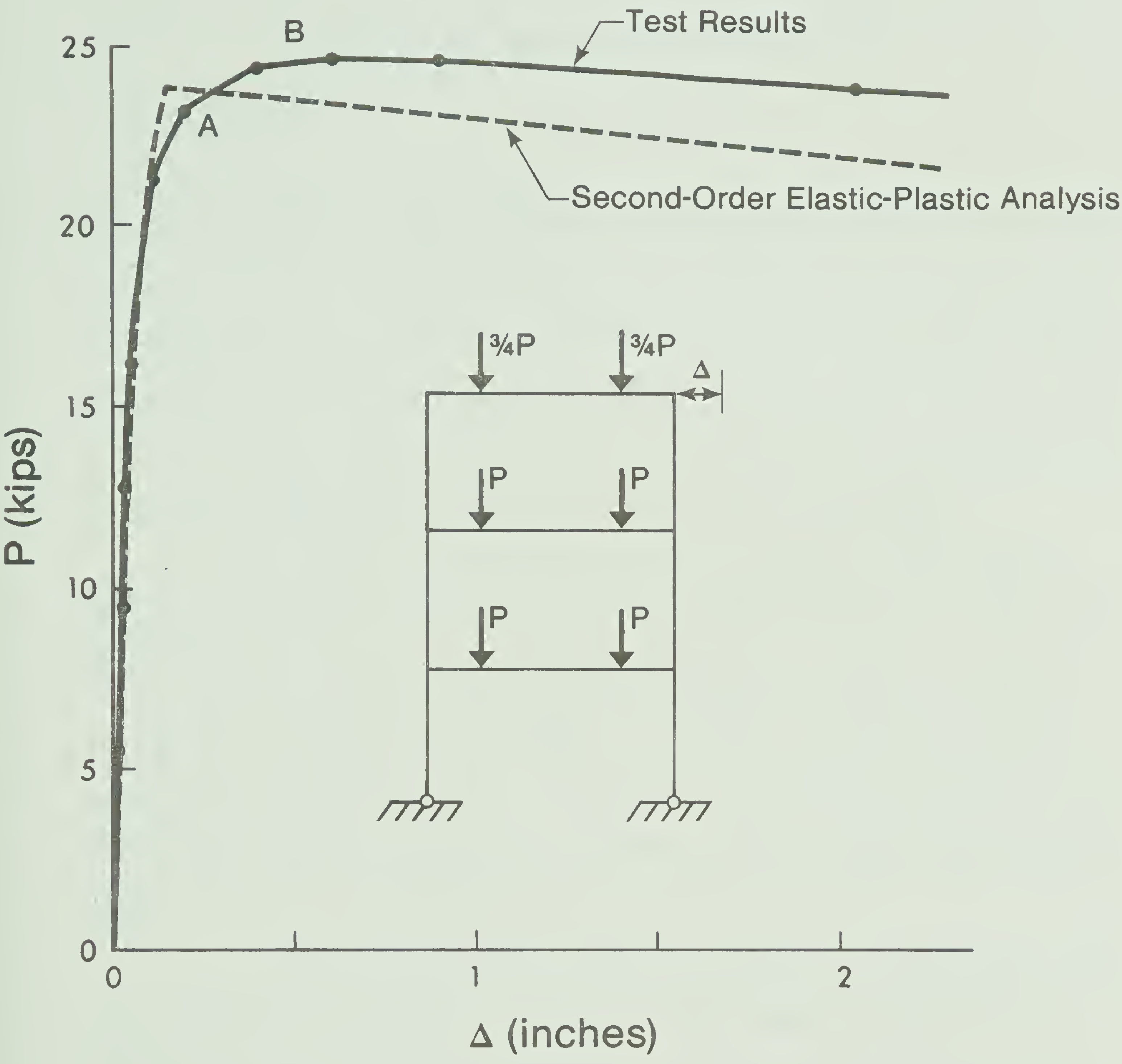


Figure 2.5 Load-Deflection Relationship, Vertical Loads Only, Three Storey Rigid Frame

CHAPTER III

REVIEW OF THE TRADITIONAL DESIGN PROCEDURE

3.1 INTRODUCTION

In the previous chapter, the various methods for the analysis of planar frames were briefly presented. At present, the analysis used for the design of a structure is generally a first order elastic analysis. The bending moments obtained for the beams are then used directly to design the girders, while the axial forces and moments for the columns are used in the interaction equations to check the column design.

As shown in the previous chapter, a first order analysis overestimates the stiffness and strength of the structure, whether the frame contains a stiff vertical bracing element or depends on member stiffness to resist lateral movement. In order to compensate for the apparent discrepancy between "true" behavior and behavior predicted by a first order analysis, the member selection has been empirically adjusted (10).

In this chapter, the traditional method of compensation for second order effects shall be reviewed. The limits obtained from the traditional method will then be compared to the results of an ultimate strength analysis of restrained beam columns permitted to sway.

3.2 ULTIMATE STRENGTH OF RESTRAINED BEAM COLUMNS PERMITTED TO SWAY

The model used to determine the ultimate strength of a beam column is based on the subassemblage technique (13). A subassemblage is

an isolated portion of a large frame that reflects the behavior of the frame as a whole. For example, in the case of an unbraced frame, the frame will deform as shown in Figure 3.1a.

To determine the strength of a column in a frame, it may be convenient to isolate the column segment and its neighboring members from the frame and treat this beam column subassemblage as a unit (13). Figure 3.1b shows a subassemblage that represents the structural action of column AB in the frame. In this subassemblage, the effects of members excluded from the subassemblage itself are represented by springs.

When studying the behavior of column AB in the subassemblage, the effect of adjacent beams and columns may also be represented by springs to obtain a restrained column permitted to sway as shown in Figure 3.1c (13).

In multi-storey buildings, the stiffness does not vary appreciably from storey to storey, especially in the storeys removed from the top and bottom of the building. Thus, in a swayed position, the columns are bent in double curvature with a point of inflection near the mid-height of the column. The behavior of a column may then be represented by a simpler subassemblage in which a pin is placed at mid-height in the column as shown in Figure 3.2a.

The forces acting on this subassemblage and the deformed configuration of the column are shown in Figure 3.2b (13). In this Figure

M_n = moment at the top of the column

M_r = the restraining moment of the beams

M_{n-1} = the moment in the column in the storey above

Pn = axial load transferred to the column top

Qn = horizontal shear carried by this column

Δn = storey deflection

From statics, the moment at the upper end of the column is

$$Mn = -\ (Qn)\ \frac{h}{2} - Pn\ \frac{\Delta n}{2} \qquad (3.1)$$

Where Qn is the lateral resistance of the column and is related to the lateral load applied to the storey, Hn, by the distribution factor, λ, in the equation

$$Qn = \lambda\ \Sigma\ Hn \qquad (3.2)$$

Using equation 3.1, the moment at the top of the column Mn may be compared to the moment in the column above, Mn-1. Since Σ Hn > Σ Hn-1, Pn > Pn-1 and λ can be assumed to be equal above and below the floor (relative stiffness of the members above and below the floor are nearly equal), then Mn-1 will likely be less than Mn (13). In fact, if the deflection Δn is greater to or equal than Δn-1, Mn will always be numerically larger than Mn-1. Therefore, Mn-1 may be conservatively considered equal to Mn (13).

Equilibrium of moments of the upper joint requires

$$Mr = -\ Mn\text{-}1 - Mn \qquad (3.3)$$

but

$$Mn = Mn\text{-}1 \qquad (3.4)$$

therefore,

$$Mr = -\ 2Mn \qquad (3.5)$$

By compatability

$$\frac{\Delta n}{h} = \theta - \gamma \qquad (3.6)$$

the shear equilibrium equation is given by

$$- M + P \frac{\Delta}{2} + \frac{Qh}{2} = 0 \tag{3.7}$$

or non-dimensionally (13)

$$\frac{Qh}{2Mpc} = - \frac{M}{Mpc} + \frac{P\Delta}{Mpc} = - \frac{M}{Mpc} + \frac{(\frac{P}{Py})(\frac{h}{r})(\frac{d}{2r})(\frac{\Delta}{h})}{2.36\ f(1-P/Py)} \tag{3.8}$$

however, for most wide-flange shapes (13)

$$\frac{d}{2r} = 1.15 \tag{3.9}$$

and $$f = 1.11 \tag{3.10}$$

therefore,

$$\frac{Qh}{2Mpc} = - \frac{M}{Mpc} + \frac{(\frac{P}{Py})(\frac{h}{r})(\frac{\Delta}{h})}{2.28(1-P/Py)} \tag{3.11}$$

The behavior of a frame may then be solved for a given $\frac{h}{r}$, $\frac{P}{Py}$ and restraining function, Mr. The restraining function, the effective resistance to rotation provided by the beam to the column is given by (35)

$$Mr = K \frac{E\ I_G}{L_G\ Mpc} \quad \theta Mpc \tag{3.12}$$

where K = restraining coefficient

K = 6 for a beam for which end rotations are equal

K = 3 for a beam pinned at the far end.

For the purpose of this analysis, beams shall be assumed to be elastic and fixed at both ends. Thus, if there are two beams framed into a column

$$Mr = \frac{6\,\Sigma}{Mpc}\left(\frac{E\,I_G}{L_G}\right) \theta\ Mpc \tag{3.13}$$

however

$$Mpc = 1.18\ Mp\ (1-P/Py) \tag{3.14}$$

where

Mp is the plastic moment for the column, $\sigma_y Z$, therefore,

$$Mp = \sigma_y Z = f\,\frac{Ic\sigma y}{d/2} = 1.11\,\frac{Ic}{d/2}\,\sigma_y \tag{3.15}$$

Substituting equations 3.14, 3.15 into 3.13 and multiplying top and bottom by h and r, the following expression is obtained for the resisting moment.

$$Mr = \frac{6E\left(\Sigma \frac{I_G}{L_G}\right)\frac{d}{r}}{1.18\ (1.11)\ \left(\frac{2Ic}{h}\right)\left(\frac{h}{r}\right)\ \sigma y\ (1-P/Py)}\ \theta\ Mpc \tag{3.16}$$

This expression for the resisting moment can be further simplified by using equation 3.9 and noting the boundary condition term used in evaluating the effective length factor for columns in multi-storey buildings, G, is given by (1)

$$G = \frac{\Sigma\ I_C/L_C}{\Sigma\ I_G/L_G} = \frac{2\ I_C/h}{\Sigma\ I_G/L_G}\ . \tag{3.17}$$

The resulting expression for the resisting moment of the beam in terms of h/r, G, P/Py, σy/E, reduced plastic moment for the columns, Mpc and joint rotation, θ, is given by equation 3.18.

$$Mr = \frac{5.27}{G\,\frac{h}{r}\,\frac{\sigma y}{E}\,\left(1 - \frac{P}{Py}\right)}\ \theta\ Mpc. \tag{3.18}$$

As a result, the ultimate strength behavior of a beam column

may be predicted given column slenderness, h/r, axial load, P/Py, material properties, $\sigma y/E$, and relative stiffnesses of members at the restraining joint, G. An example calculation is given in Table 3.1 and results plotted in Figure 3.3.

First, a series of joint rotations, θ, are assumed and the corresponding restraining moments Mr are calculated. The corresponding column end rotation, γ, can then be obtained from column moment-rotation charts (13). The deflections are then obtained using equation 3.6. Finally, the shear equilibrium equation (equation 3.8) is used to solve for the shear resistance of the column in terms of the column moment, M/Mpc and the second order shears, $P\Delta$/Mpc. If this process is repeated for a number of values of joint rotation, θ, a full load-deformation relationship for a given column with a constant axial load can be obtained.

Plotted in Figure 3.3 are the load-deflection curve and the column end moment-deflection curve for a given column and constant axial load. The difference between these curves is the $P\Delta$ term. This type of analysis has been compared to experimental results and found to give a reasonable prediction of a restrained column permitted to sway (21,22).

3.3 DESIGN OF BEAM COLUMNS USING INTERACTION EQUATIONS

At present, for steel structures in Canada, a designer has the option of using one of two standards (40). CSA S16-1969 permits the designer to use allowable stress design for all structures and permits plastic design of portions of certain structures (2). CSA S16.1-1974 permits the use of Limit States Design of all aspects of steel structures (24). For the purpose of this dissertation, the design of beam-columns

shall be outlined using the interaction equation as specified in CSA S16.1-1974. The basis for the development of the interaction equations in each standard, however, remains essentially the same (43).

In order to comply with the requirements of CSA S16.1-1974, class 1 or class 2, sections of I-shaped members subjected to combined axial load and bending moment must satisfy three equations (24). The first equation satisfies the requirement that the beam shall not fail in pure biaxial bending by ensuring that:

$$\frac{Mfx}{Mrx} + \frac{Mfy}{Mry} \leq 1.0 \tag{3.19}$$

where, Mfx and Mfy are the bending moments about the x and y axis of the member respectively under factored load and Mrx and Mry are the factored moment resistance of the member about the x and y axis of the member respectively.

The member is then checked for sufficient strength to resist local failure due to combined axial force and bending in the following equation:

$$\frac{Cf}{Cr} + \frac{0.85\ Mfx}{Mrx} + \frac{0.60\ Mfy}{Mry} \leq 1.0 \tag{3.20}$$

where:

Cf = the compressive force in the member under factored load; factored axial load

Cr = factored yield load for the member.

Finally, the member must be checked for failure by overall instability. This condition is checked by using the third interaction

equation:

$$\frac{Cf}{Cr} + \frac{\omega x \ Mfx}{Mrx(1 - \frac{Cf}{Cex})} + \frac{\omega y \ Mfy}{Mry(1 - \frac{Cf}{Cex})} \leq 1.0 \qquad (3.21)$$

where, in this case:

Cr = factored compressive resistance of the member if axial load only is present,

Ce = Euler buckling load = $286{,}000A/(KL/r)^2$ where $(\frac{KL}{r})$ represents the effective slenderness ratio in the plane of bending and A is the cross-sectional area,

ω = the coeefficent used to determine the equivalend bending effect in steel columns.

If the primary bending moment is not uniform over the member length, the strength of the column will be increased. To account for this strength increase, an equivalent moment factor, ω, is computed by choosing one of the following formulae:

a)

$$\omega = 0.6 + 0.4 \frac{M_{f1}}{M_{f2}} \text{ but not less than } 0.4 \qquad (3.22)$$

where, M_{f1} and M_{f2} are the larger and smaller moments respectively acting at the ends of the member. M_{f1} and M_{f2} are both positive if the member is deformed into single curvature and M_{f1} is negative if the member is in double curvature.

b)

$$\omega = 0.85 \qquad (3.23a)$$

for members bent in double curvature or subject to moment at one end.

or

$$\omega = 1.00 \tag{3.23b}$$

for members bent in single curvature due to moments at each end.

Traditionally, the choice between case a (equation 3.22) and case b (equation 3.23a or 3.23b) was based on the manner in which lateral forces were resisted by the frames (2). If the lateral forces applied to the frame were resisted by a stiff vertical bracing system, the frame was classified as sway prevented and equation 3.22 would be used to calculate ω. If, however, only the flexural stiffness of the compression members was used to resist lateral forces, the frame is classified as sway permitted and either equation 3.23a or equation 3.23b is used.

In the stability interaction equation (equation 3.21), the term $1/(1 - Cf/Ce)$ is an amplification factor that accounts for the secondary moments produced by axial loads acting on the deformed member. The effect of boundary conditions at the member ends are included in the calculation of the effective length of the member, KL, which in turn is used to calculate the term Ce in the amplification factor. As a result, the calculation of the effective length has a direct influence on the magnification of the moments in the member to account for second order effects. In addition to its use in moment magnification, the effective length factor is also used in the calculation of the factored compressive resistance of the column, Cr.

The physical significance of the effective length of a member can only be related to axially loaded members. The effective length of a member is the distance between theoretical points of inflection of

the deflected shape of a member at the point of buckling (31). Thus, the calculation of the effective length for a member depends on whether the frame is assumed to buckle in a sway prevented mode or in a sway permitted mode (31).

A procedure for evaluating the effective length factor, K, for both sway permitted and sway prevented frames was developed by Julian and Lawrence (48), and is outlined by Galambos (1). The effective length factor is calculated in terms of the following parameter:

$$G = \frac{\Sigma\ I_C/L_C}{\Sigma\ I_G/L_G} \quad . \tag{3.24}$$

In the sway permitted case, the effective length factor, K, is obtained by the solution of (1)

$$\frac{G_L\ G_U\ (\pi/K)^2 - 36}{6\ (G_L + G_U)} = \frac{(\pi/K)}{\mathrm{Tan}(\pi/K)} \tag{3.25}$$

In this equation, the L and U subscripts refer to the upper and lower end of the column respectively. In the sway prevented case, the following equation is used (1)

$$\frac{G_U\ G_L}{4}\ (\pi/K)^2 + \frac{(G_U + G_L)}{2}\ \frac{1 - \pi/K}{\mathrm{Tan}(\pi/K)}$$

$$+ \frac{2\ \mathrm{Tan}\ (\pi/2K)}{\pi/K} = 1 \tag{3.26}$$

As a result, to calculate the effective length of the member when using equation 3.22, the frame must be first classified as "sway prevented" or "sway permitted". Traditionally, this classification has

been made on the basis of the method used in the structure to resist lateral loads, that is, whether a stiff vertical bracing element is included in the frame (2).

3.4 COMPENSATION FOR SECOND ORDER EFFECTS

The manner in which a compensation is made for the second order effects in the traditional design method is illustrated by considering the behavior of a restrained column permitted to sway as outlined in Section 3.2.

The example column is assumed to have a yield strength of 36 ksi, a slenderness ration, $h/r = 40$, and is restrained top and bottom by elastic beams so that G (Equation 3.24) is equal to 2. The model used for analysis is shown as an inset to Figure 3.4. The interaction between axial load and resisting moment of the column at ultimate strength can be predicted by using the analysis developed in Section 3.2 and calculating the resistance of the column to lateral load for a given axial load. The axial load applied is non-dimensionalized as P/Py where Py is the yield load of the column. The resistance of the column to lateral load at ultimate strength is non-dimensionalized as $Qh/2Mp$ where Mp is the plastic moment of the column. This relationship is shown as the lower solid line in Figure 3.4.

In addition, the interaction between the moment at the top of the column at ultimate load, $M_T/M_P = Qh/2Mp + P\Delta/ Mp$, and the axial in the column is shown as upper solid line. The difference between these curves at any specific value of axial load is the second order moment at the ultimate load.

The ultimate strength predicted by the interaction equations is the envelope of the combined interaction equations. For the case of a member classified as sway prevented, this envelope is shown as the upper dashed line in Figure 3.4 as bounded by Equation 3.19, Equation 3.20, and equation 3.21(ω and K based on the sway prevented model). This limit is a reasonable prediction of the maximum moment in the member at ultimate strength (upper solid curve). If the second order effects are effectively resisted by some other members in the structure such as a bracing system or a shear wall, then the interaction equations give a realistic prediction of beam column behavior.

If, however, the frame has been classified as sway permitted, the ultimate strength predicted by the interaction equations is the envelope shown as the lower broken line in Figure 3.4 bounded by equation 3.21 with ω and K based on the sway prevented model along with equations 3.20 and 3.21. This envelope gives a somewhat conservative prediction of the ultimate strength interaction of a restrained column permitted to sway as shown by the lower solid curve in Figure 3.4.

Thus, for a given value of axial load, P/P_y, if a structure is classified as sway permitted, the ultimate moment for the column is limited by lower dashed curve. For example, for $P/P_y = .6$, $M/M_p = 0.26$. The ultimate strength of the beam column is predicted by the upper dashed curve, $M/M_p = .47$. The difference between the limiting moment for a sway permitted column and the ultimate moment of the column is the compensation for second order moments when using the interaction equations. This compensation is based on the classification of the frame as sway permitted or sway prevented and the resulting differences in the equiva-

lent moment factor, ω, and the effective length factor, K, used in equation 3.21.

A further comparison of the ultimate strength of restrained columns permitted to sway and the limit predicted by equation 3.21 with ω and K based on the sway permitted model is shown in Figures 3.5, 3.6, and 3.7 for various values of G and h/r. In general, the interaction equation is a conservative prediction of the ultimate strength of the column. This result confirms an analysis by Yura and Galambos for single storey frames (26).

3.5 SUMMARY

While the results of a first order analysis may be used directly in the interaction equations, the second order effects are indirectly compensated for in the design by use of a "sway permitted" model for calculation of certain terms in the stability interaction equation. This approach to beam column design has been confirmed to be at least conservative in this chapter and in previous studies (26). The somewhat conservative nature of the traditional method for design of beam columns has lead to proposals to change this design method (4,5,6,7).

However, there has been no attempt to compensate for second order effects when the design is controlled by member strength (Equation 3.20 and Equation 3.21) or to compensate for second order effects in beams.

TABLE 3.1

ANALYSIS OF RESTRAINED COLUMN PERMITTED TO SWAY

$G = 2.0$ $H/R = 40$ $\sigma y = 36$ $P/Py = 0.60$

θ (Assumed)	MR/MPC (EQ. 3.18)	M/MPC (EQ. 3.5)	γ (M-γcurve)	Δ/H (EQ. 3.6)	QH/MPC (EQ. 3.11)
0.00373	1.0	-0.50	-0.0019	0.00563	0.353
0.00447	1.2	-0.60	-0.0023	0.00677	0.423
0.00522	1.4	-0.70	-0.0028	0.00802	0.491
0.00596	1.6	-0.80	-0.0033	0.00926	0.558
0.00634	1.7	-0.85	-0.0038	0.01014	0.585
0.00671	1.8	-0.90	-0.0043	0.01101	0.613
0.00690	1.85	-0.925	-0.0048	0.01170	0.620
0.00697	1.87	-0.935	-0.0050	0.01197	0.623
0.00704	1.89	-0.945	-0.0052	0.01224	0.625
0.00708	1.90	-0.950	-0.0053	0.01238	0.627
0.00712	1.91	-0.955	-0.00554	0.01266	0.625
0.00745	2.00	-1.000	-0.00910	0.01655	0.568

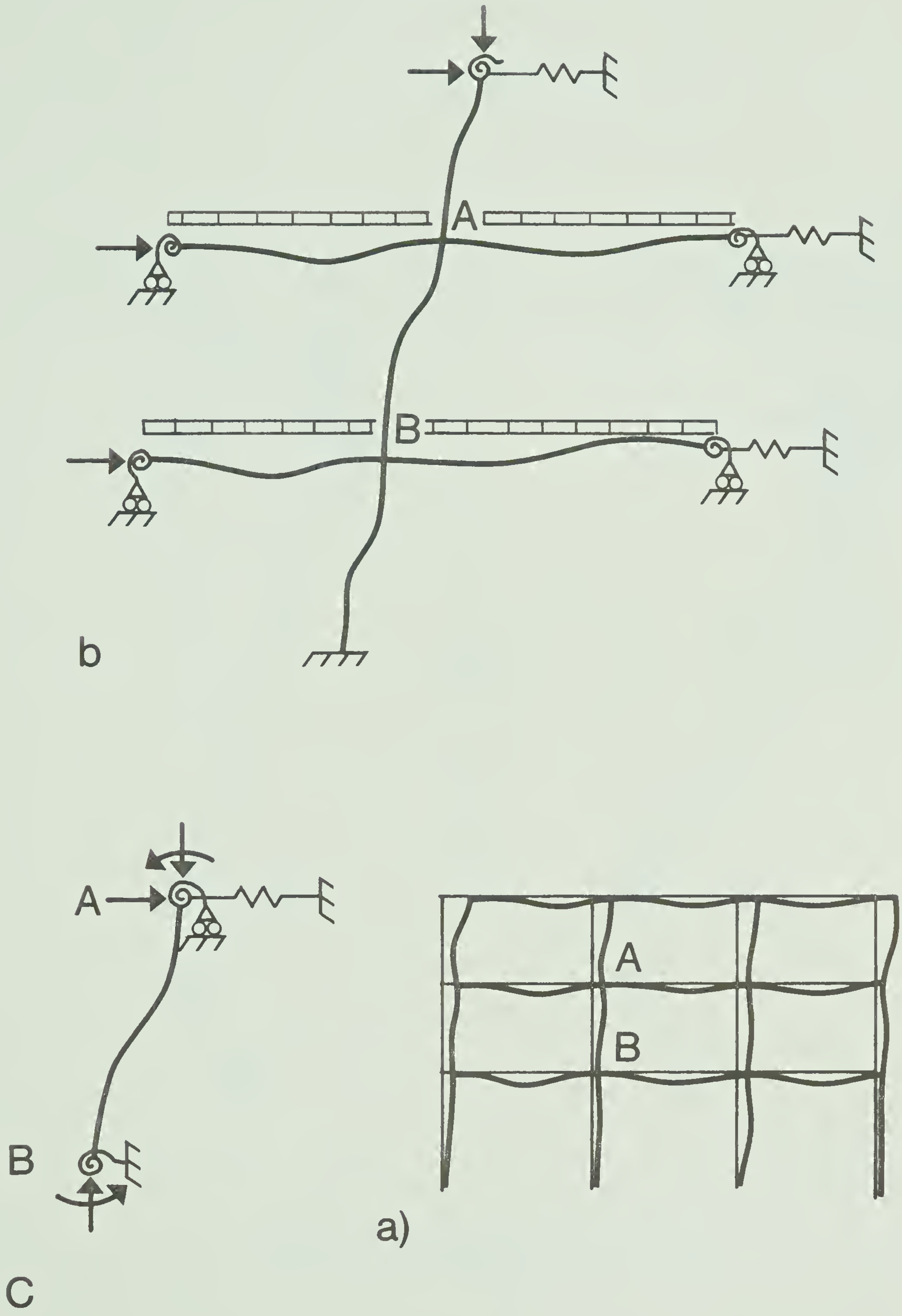


Figure 3.1 Unbraced Frame, Sway Subassemblage and Restrained Column

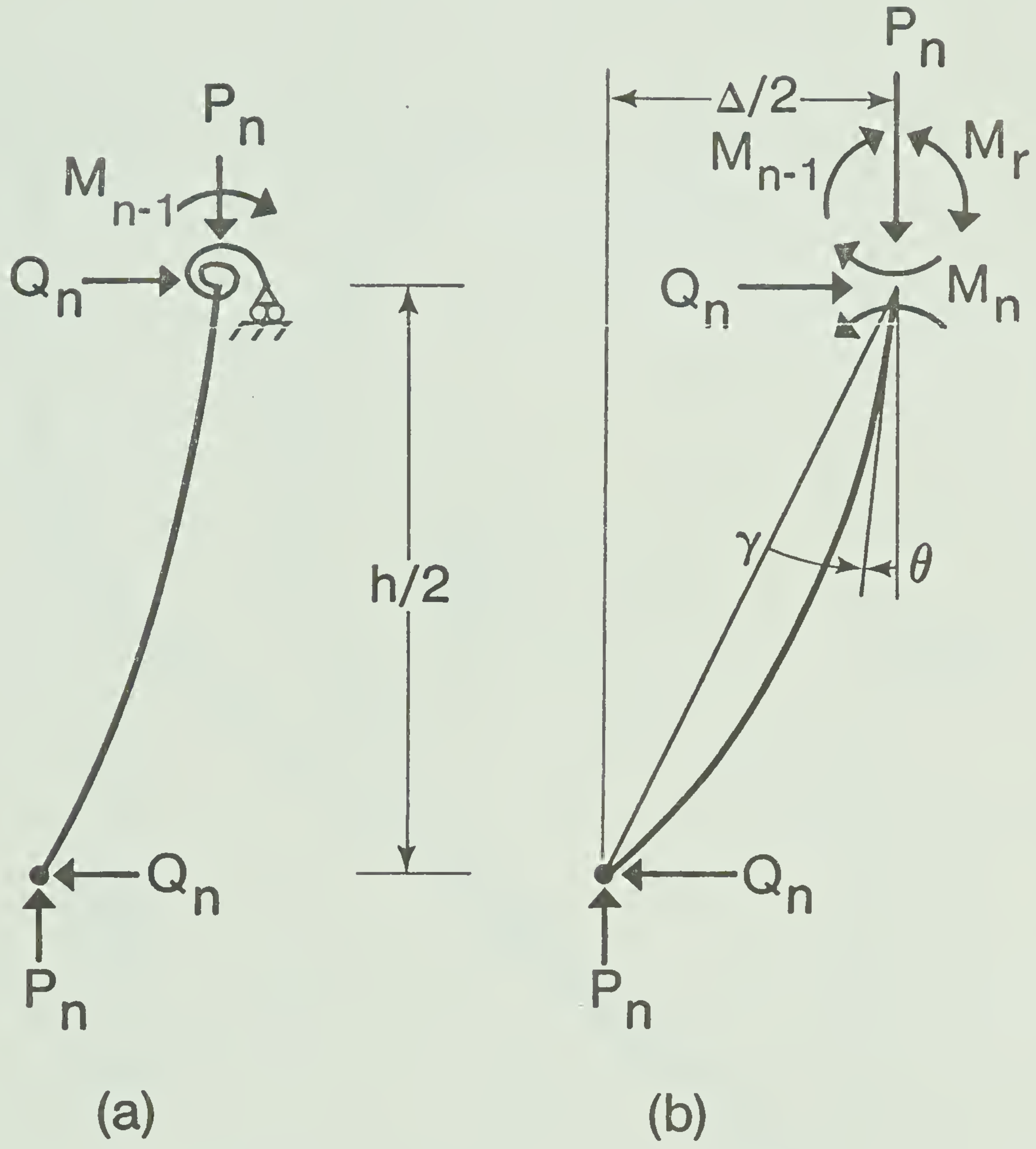


Figure 3.2 Restrained Column Model

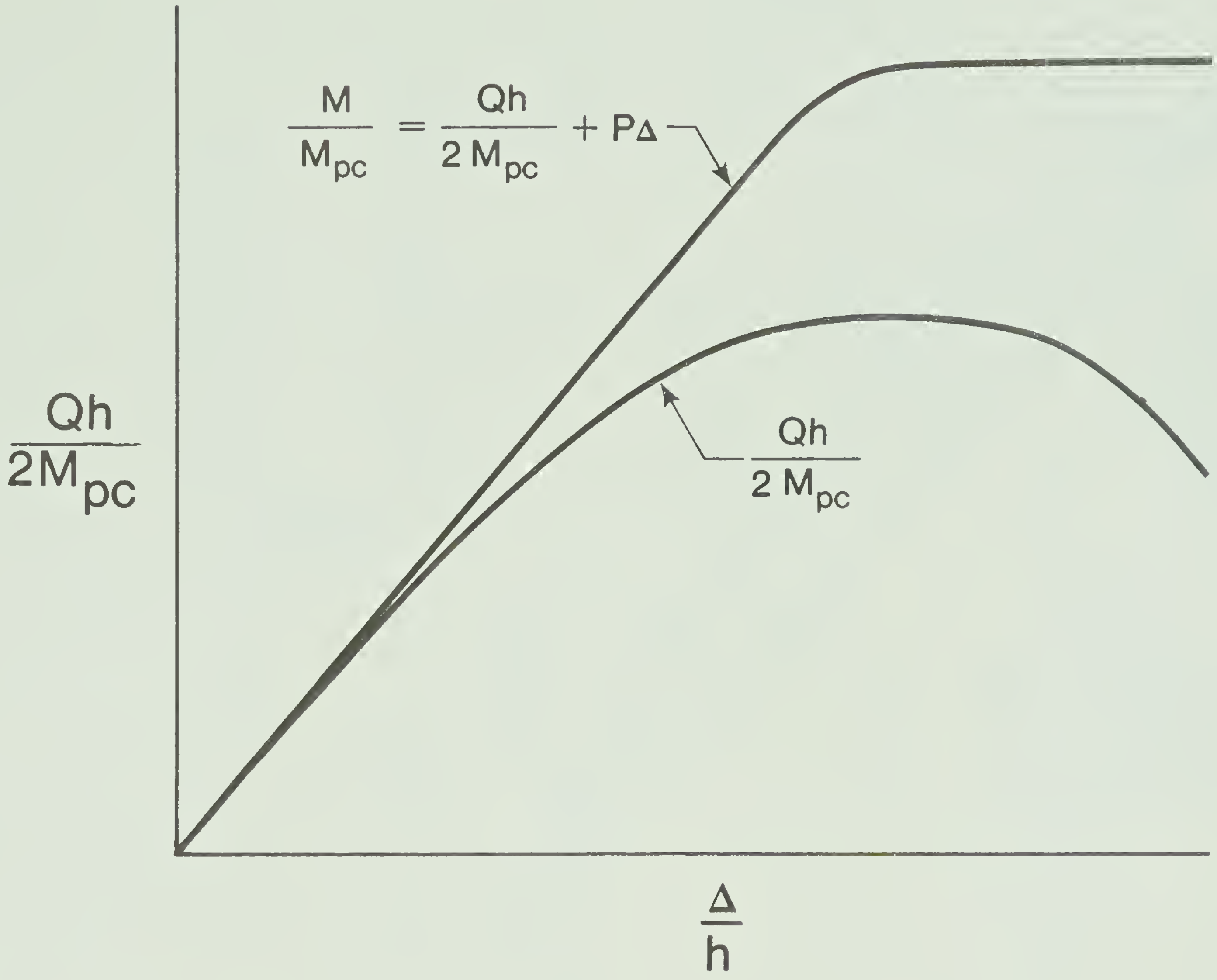
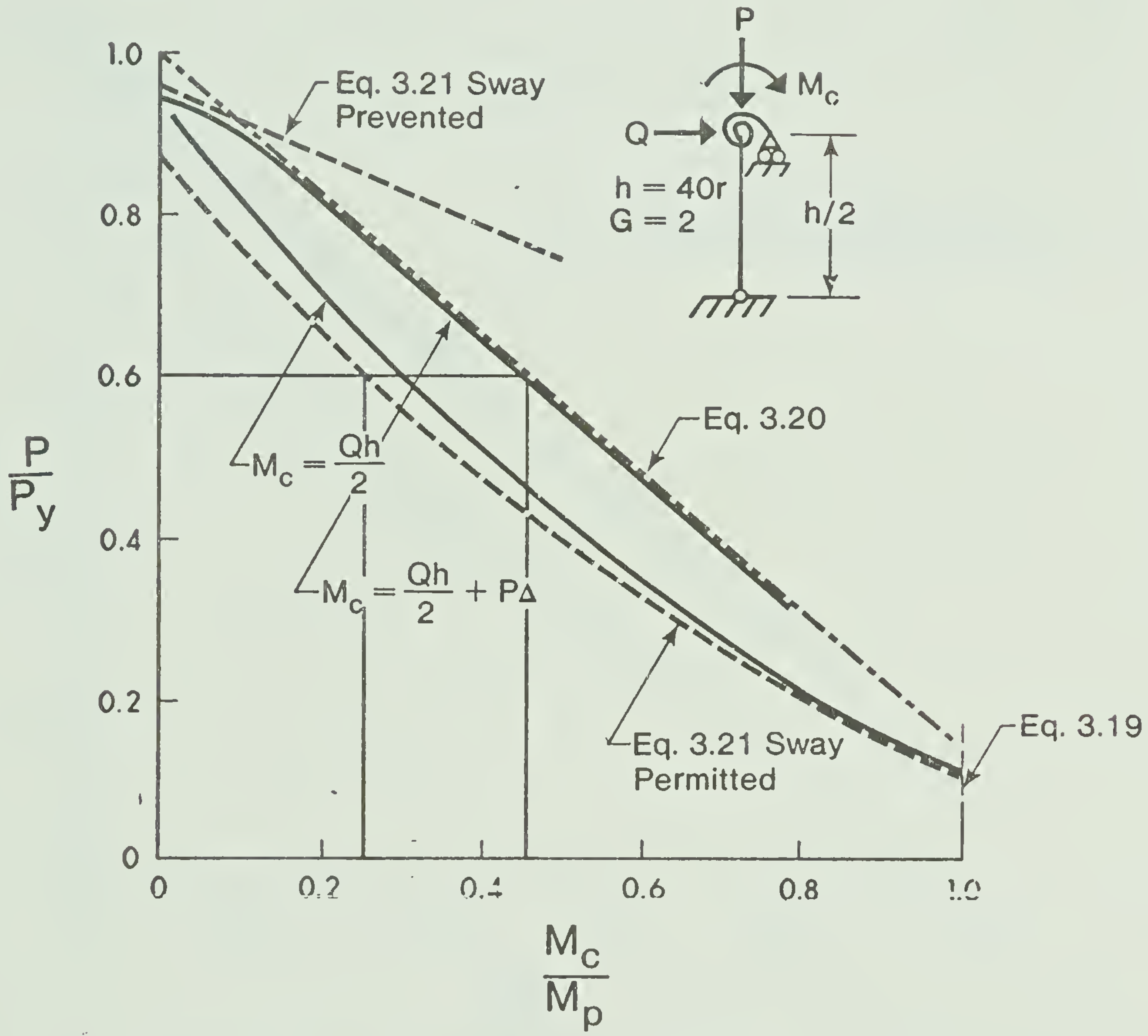


Figure 3.3 Column Moment and Sway Resistance vs Sway



Figue 3.4 Interaction Relationship, H/R = 40, G = 2

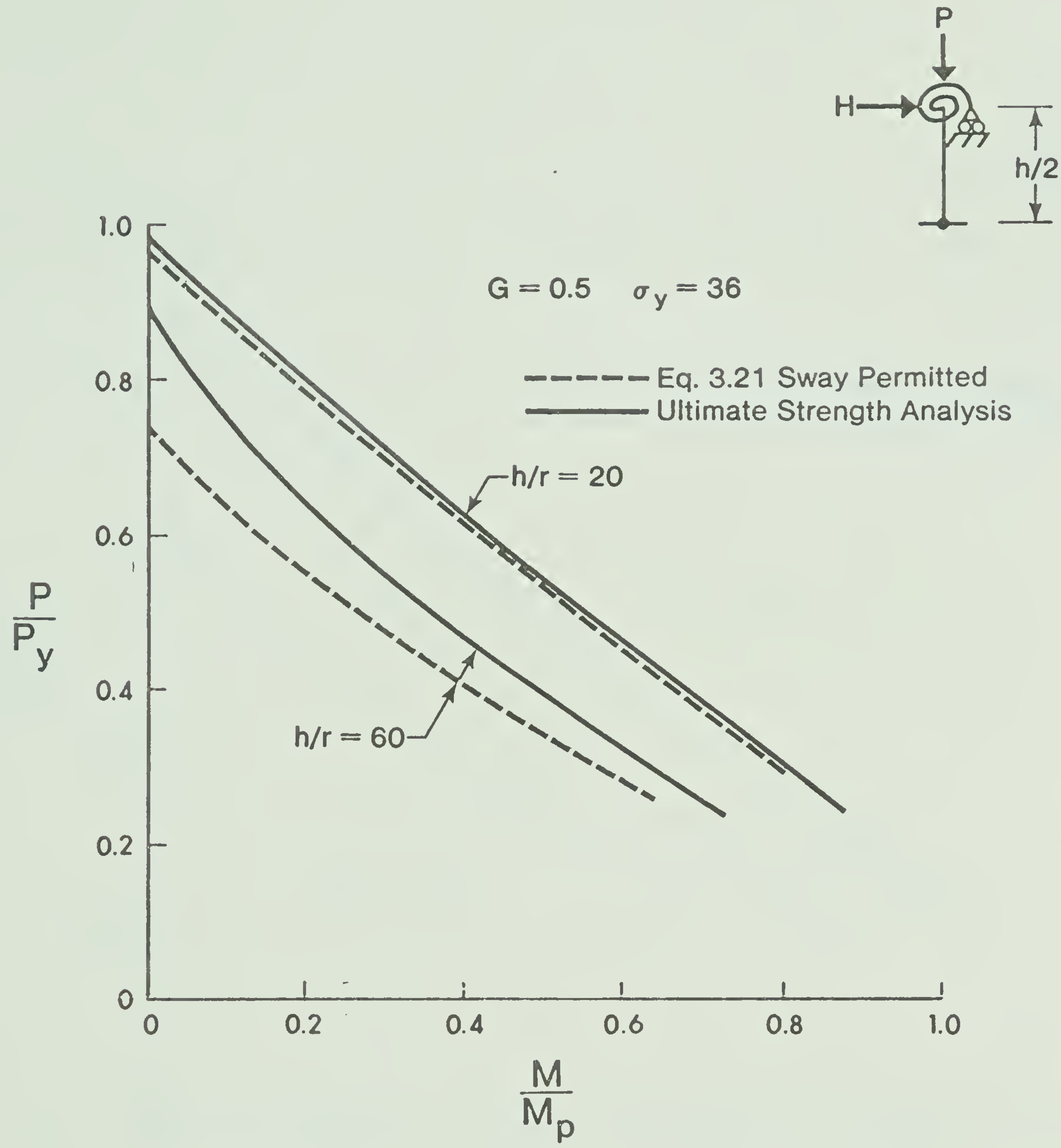


Figure 3.5 Interaction Predictions, G = 0.5

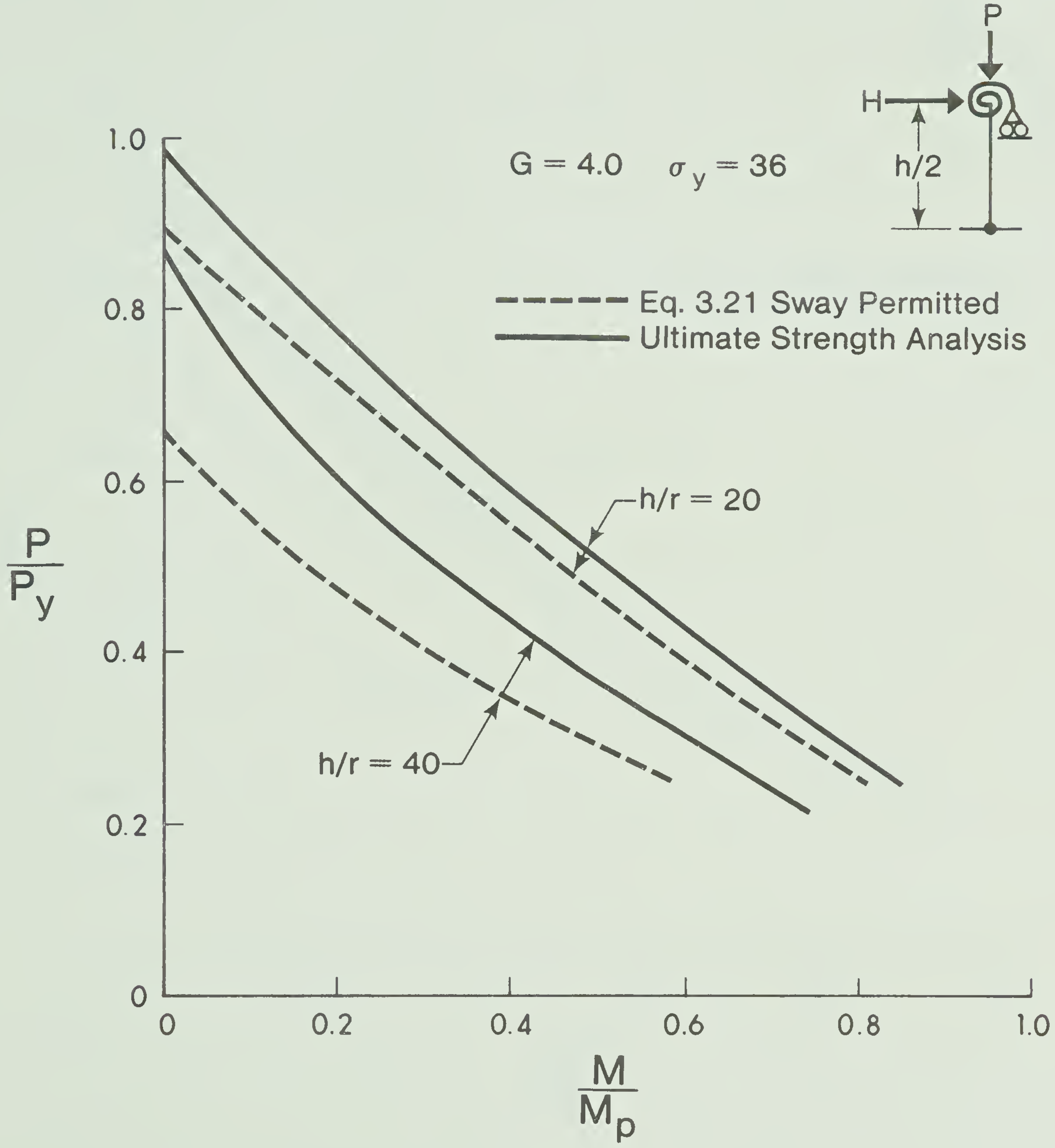


Figure 3.6 Interaction Predictions, G = 4.0

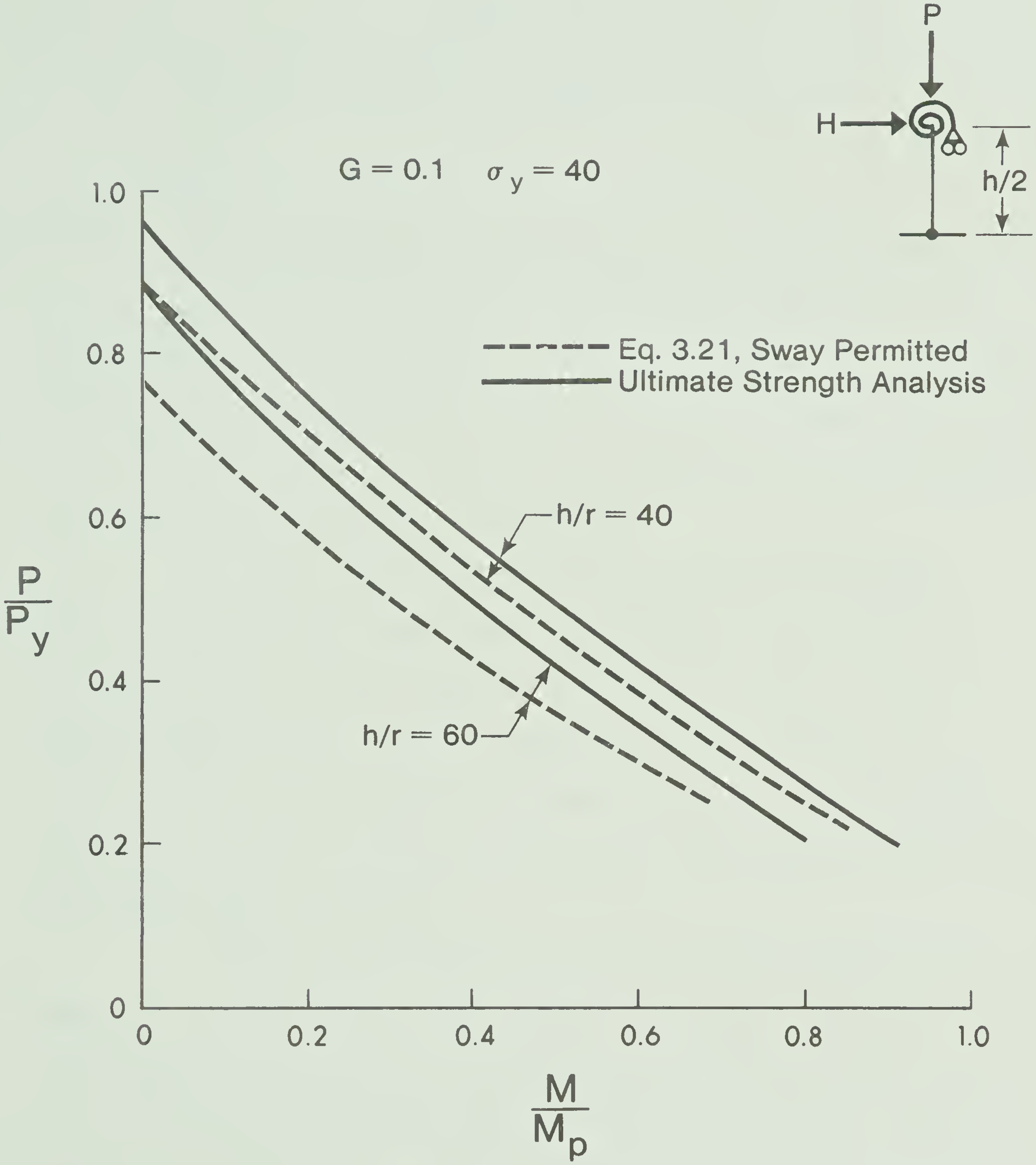


Figure 3.7 Interaction Prediction G = 0.1

CHAPTER IV

PROPOSED MODIFICATIONS TO THE TRADITIONAL METHOD

4.1 INTRODUCTION

In the previous chapter, the use of the effective length factor, K, along with the equivalent bending moment coefficient, ω, to compensate for second order effects, has been discussed. Most criticisms of the traditional method for the design of beam-columns have been based not on the use of K and ω in the stability interaction equation (Equation 3.21), but on the model used to calculate K(4).

The traditional method for the calculation of K assumes that all columns of the frame buckle simultaneously, and effectively, the columns cannot provide support to each other (1). As a result, a number of modifications have been suggested that would consider the lateral restraint provided by other columns of the frame (4,6) or the rotational restraint that could be provided by other columns framed into the column in question (6,30).

Yura has questioned the use of only elastic properties of members in the calculation of the effective length of a column (7). Yura proposed a method for the calculation of an effective length based on a model that would include inelastic behavior of columns. Yura has also proposed a method of considering the leaning action of a flexible frame on a more rigid frame through a modification of the effective length factor (7).

The object of this chapter is to outline these proposed mo-

difications to the traditional method of beam column design and discuss the implications of these proposals with respect to the stability effects in building frames.

4.2 K FACTORS DETERMINED BY FRAME BUCKLING ANALYSES

The effective length concept relates the buckling load of a member with given end conditions to the buckling load of the same member with ends effectively pinned and translation prevented by the equation

$$K^2 = \frac{Pe}{Pcr} \tag{4.1}$$

where K is the effective length factor

$Pe = \frac{\pi^2 EI}{L^2}$ is the Euler buckling load

Pcr is the buckling load of the column in question.

The problem, when determining the K factor for a given column in a multi-storey building frame, is to relate the effect of other members of the frame to the stability of the column in question.

At the time the traditional method was developed for calculation of K factors, the buckling solution of a large frame was too complex for available computation facilities (1). The traditional model used to determine effective length factors was based on a series of simplifying assumptions.

The columns of the frame were assumed to buckle simultaneously (1), thus, the column in question could not be supported laterally by other columns in the frame. In addition, the restraining moments of

the beams were assumed to be distributed between the column in question and the other column framed into the joint in the ratio of the I/L values of the two columns (1). The implications of these assumptions have been the focus of many of the proposals to modify the calculation of the effective length factor, K (4,6,50).

With the advent of more sophisticated electronic digital computers, buckling solutions of larger structural systems have been developed. Haldorson and Wang (29), developed a program for the stability analysis of frameworks. To determine the buckling loads for a structural framework, the effects of axial loads are included in the formulation of member stiffness resulting in a stability stiffness matrix for the structure, [K], (29).

For a structural system whose members are subjected to axial loads only, neglecting axial deformations, the joint loads relative to the unrestrained joint deformations are zero. That is, at bifurcation, only the critical primary axial forces are capable of maintaining equilibrium and compatible deformation without external forces (42). In matrix form

$$\{P\} = [K]\,\{X\} = 0 \tag{4.2}$$

where {P} are the primary axial forces

{X} are the unrestrained joint deformations

[K] is the stability stiffness matrix.

For the non-trivial solution, the determinant of the stability matrix, [K], must be zero (29). For a structure with an axial loading system, the axial loads may be varied by an axial load factor, N, resulting in a variation to the stability stiffness matrix [K]. The

critical load factor, N_{cr}, may be determined by varying the load factor by increments, checking the value of the determinant of [K]. The value of N_{cr} lies between increments of N for which the determinant of [K] changes from positive to negative in sign (42).

The effective length factor for each axially loaded member of the frame can then be calculated based on the critical buckling load factor for the frame. For example, for the n^{th} member of the frame, given the applied axial load for that member, P_n, and the Euler buckling load for that member P_e, the effective length of the member, K_n, is given by

$$K_n = \sqrt{\frac{P_e}{N_{cr} P_n}} \qquad (4.3)$$

While Holdorson and Wang used a frame buckling analysis to determine effective length factors of axially loaded members, they did question the use of these effective length factors in the determination of allowable stresses (29). They suggested, however, that the effective lengths used in present specifications were defective because the effect of the relative values of axial loads on the effective length ratios were not included. In conclusion, Holdorson and Wang believed that the incorporation of the ultimate strength theory with a second order structural analysis beyond the elastic range was perhaps a more logical approach upon which approximate methods of design practice should be based (29).

An approximate method for the determination of frame buckling loads was developed by Edmonds and Medland (6). The method involved the determination of a "reserve lateral stiffness" for the columns of a

storey (6). The critical load factor for a storey is determined when this reserve lateral stiffness becomes zero. The critical load factor for the frame is the smallest of the critical load factors determined for the stories. The critical load factor was then used to determine effective length factors based on frame buckling (6). Edmonds and Medland concluded that columns proportioned using traditional methods to determine the K factor were seriously under-designed when compared to those proportioned using the overall frame critical load factor to establish the effective length (6).

To examine the implications of the use of effective length factors based on frame buckling for the design of beam columns, an analysis of a ten-storey frame was performed using the method proposed by Edmonds and Medland (6). The frame used in the analysis was the ten-storey frame designed using an allowable stress method in the Lehigh Lecture Notes (13). The member sizes for the frame are shown in Figure 4.1. The results of the analysis are shown in Table 4.1. In this Table, and for the purpose of this discussion, the effective length factors based on a solution of Equation 3.25 are designated as K_T, the effective length factors based on the buckling load factor for the total frame are designated as K_F and effective length factors based on the buckling load factor of each storey are designated as K_S. The buckling load factors for each storey are also shown in Table 4.1 as λc.

A comparison between the traditional effective length factors, K_T, and the effective length factors based on frame buckling, K_F, shows that in nearly all cases, for a given column, K_T, is greater than, K_F. This discrepancy is particularly significant in the stories near the top

of the frame. The significance of the difference between K factors obtained by each method can be obtained by plotting the stability interaction equation (Equation 3.21) using each K factor. The limits obtained can then be compared to an ultimate strength interaction prediction as outlined in Chapter 3.

The interaction between axial load, P/Py and ultimate moment M/Mp, predicted by series of ultimate strength analyses of column 1 of storey nine of the frame is shown as the solid line in Figure 4.2. The interaction predicted by Equation 3.21 with ω = .85 and K = K_T is shown as the broken line in Figure 4.2. The interaction predicted by Equation 3.21 with ω = .85 and K = K_F is shown as the dashed line in Figure 4.2. The results of similar analyses of column 2 of storey nine are shown in Figure 4.3.

In each case, the interaction limit predicted by Equation 3.21 with K = K_T is slightly conservative when compared to the results predicted by the ultimate strength analysis. If, however, K = K_F is used in Equation 3.21, the interaction limit predicted is considerably more conservative when compared to the results of the ultimate strength analysis.

The conservative nature of the limits obtained when an effective length factor based on frame buckling is used in Equation 3.21 arises from an improper application of this effective length rather than an incorrect derivation of the effective length. The physical interpretation of effective lengths of members of a frame only relate to frames in which members are subjected to axial loads. The effective length derived on the basis of frame buckling represents the theoretical

distance between points of inflection at the instant of buckling (31).

If there are large variations in member stiffness and loading, as may often be found in tall structures, the actual buckling of the frame becomes quite localized. This phenomena is illustrated by a comparison of the theoretical critical load for each floor in Table 4.1. For this particular frame, the variation of critical load factors from floor to floor is considerable, from 13.04 in storey two to 27.98 in storey ten. The buckling of the frame is the result of conditions that exist in storey two and adjacent stories, and is not significantly influenced in this case by conditions in the top stories of the structure.

The effect of the use of a K_F for the design of the members of this example frame would be that members in stories nine and ten would be proportioned for conditions that exist in storey two of the frame. There, however, is no relationship between the second order effects that occur in stories nine and ten and the critical buckling of storey two of the frame. Thus, the use of a K factor based on the critical buckling of the frame would result in even more conservative design and does not reflect a more rational approach to compensation for second order effects in the frame.

Effective lengths calculated on the basis of the critical loads for each storey could be of interest. These storey buckling K factors, K_S as shown in Table 4.1, would reflect only local conditions and could reflect the supporting of more critical columns by other members in that storey.

This supporting of one member in a storey by the other members of a storey is illustrated in storey nine of the example. For

column 3 in storey nine, $K_S = 0.70$. A K factor of less than one implies the member is effectively braced against sidesway buckling. In this case, the support is provided by other columns in the storey rather than a bracing system. The result of this analysis confirms a suggestion by Lay that less critically loaded columns in a framing system can provide support to neighboring columns, in effect, preventing these columns from buckling in a sway permitted mode (5). It must be noted, however, that while the effective length factors for certain columns of the frame have been decreased as a consequence of the support provided by a less critically loaded column, there is a corresponding increase in the effective length of the supporting columns. For example, for column 1 in storey nine, $K_S = 1.77$, while $K_T = 1.68$.

To examine the use of K_S in Equation 3.21, a comparison was made of the ultimate strength interaction between P/Py and M/Mp, as predicted by the analysis of Chapter 3 and the prediction obtained using Equation 3.21 and $K = K_S$ for column 2 of storey nine. The results of these analyses are shown in Figure 4.4. The ultimate strength analysis is shown as the solid curve. Equation 3.21 with $K = K_S$ is shown as the dashed curve. For a large range of values of P/Py, the interaction limit predicted by Equations 3.21 and $K = K_S$ is unconservative. For a similar analysis of column 1 in storey nine, the interaction predicted by Equation 3.21 and $K = K_S$ is shown as conservative in Figure 4.5.

Thus, if the effective length factor was to include the influence of the members of the storey in question, in some cases, the resulting designs may be unsafe in the limit state for that individual member (24). While the reserve capacity of supporting members of the

frame, such as column 1, may result in a sufficient lateral load capacity for the frame as a whole, this implies, however, a re-distribution of load carrying capacity between members of the frame that is not permitted when using elastic design techniques (2,24).

4.3 EVALUATION OF END RESTRAINTS PROVIDED TO COLUMNS

To evaluate the buckling load for an individual column of a frame, the rotational restraint provided by other members framed into the ends of this column must be determined. If the joints of the frame are assumed to be rigid, then the end restraint provided to a column is measured by the moment developed at the joint per unit rotation of the joint. The moment developed at a joint A of a member, M_{AB}, by a rotation at joint A of θ_A, a rotation at the far end of the member of θ_B, and a relative sway of the ends of the member, ρ, is given by

$$M_{AB} = \frac{EI_{AB}}{L_{AB}} (S\theta_A + SC\theta_B - S(1 + C)\rho) \qquad (4.4)$$

where S and C are stability functions dependent on the axial load in the member, P/P_E (32).

In the case of a beam, the axial load is negligible, therefore, C = ½ and S = 4. In addition, the sway is zero. Thus, if θ_A is assumed equal to θ_B, the restraining function provided by a beam to the column is given by

$$M_{AB} = 6 \frac{EI_{AB}}{L_{AB}} \theta_A \qquad (4.5)$$

The case of an axially loaded member framing into the joint (usually a column above or below the column to be designed) is consi-

derably more complex. The restraining moment provided by Equation 4.4 depends on the axial load in the member as well as the relative movement of the joints as the member buckles. As a result, the restraining moment can have a positive or negative value. A positive value of restraining moment implies that the axially loaded member will restrain the rotation of the joint. A negative moment implies that the restraint provided by the beams at the joint must be divided between the columns above and below the joint.

To facilitate the calculation of effective length factors in complex frames with various loading conditions, all columns were assumed to buckle simultaneously (1). The resisting moments supplied by the beams were divided between columns in proportion to the relative I/L values of the columns. This assumption is reflected in the term G given by

$$G = \frac{\Sigma I_G/L_G}{\Sigma I_C/L_C} \tag{4.6}$$

used in Equations 3.25 and 3.26 to determine sway permitted and sway prevented K factors.

Lay, however, suggested that the assumptions used to derive Equation 4.6 were overly conservative. He suggested a more exact solution of the critical loads for columns or at least Equation 4.6 should be adjusted to reflect the influence of axial loads on members framing into the column in question. Lay suggested an alternative form for the calculation of G that would be preferable in all situations (30).

$$G' = \frac{\frac{I\ \text{column}}{L\ \text{column}}\ \text{being designed}}{\Sigma(1 - P/P_e)\frac{I}{L}\ \text{all other members}} \tag{4.7}$$

Lay, unfortunately, did not study the effect of using an effective length factor calculated on the basis of Equation 4.7 in the beam column interaction equations. To study the effect of the use of Equation 4.7 on the stability interaction equation (Equation 3.21), an ultimate strength analysis was performed on an elastic plastic beam column subassemblage as outlined in Chapter 3. The solid line in Figure 4.6 illustrates the interaction between axial load and ultimate moment for a subassemblage in which $h/r = 20$, $G = 2$, and $\sigma_y = 36$. To obtain a value of G' using Equation 4.7, P/P_E was assumed to be 0.1 and I_c/L_c was assumed equal for each storey. Therefore,

$$G = \frac{\Sigma\ Ic/Lc}{\Sigma\ I_B/L_B} = 4 \tag{4.8}$$

or

$$\Sigma\ I_B/L_B = .5\ \frac{Ic}{Lc}$$

Thus, in Equation 4.7

$$G' = \frac{I_c/L_c}{.5\ I_c/L_c + (1 - .1)I_c/L_c} = .71 \tag{4.9}$$

This value of G' is used in Equation 3.25 to obtain an effective length factor, K, of 1.22. Using this value for K, Equation 3.21 was plotted as the dashed line in Figure 4.6. For all values of P/P_y, the interaction predicted by Equation 3.21 and $K = 1.22$ is slightly unconservative. Similar results are shown in Figure 4.7 for $h/r = 40$,

$G = 2$, $\sigma = 36$.

Thus, the use of a revised value for end restraint provided by Equation 4.7 may lead to designs that are slightly unconservative when compared with the results of an ultimate strength interaction of a beam-column subassemblage. While Lay's suggestion may provide a more realistic value for the buckling load of a column, the use of such a K factor when designing beam columns is questionable.

4.4 EFFECT OF INELASTIC COLUMN ACTION

Another modification to the calculation of the effective length factor, K, has been proposed by Yura (7). Yura questioned the use of elastic column stiffness to evaluate effective joint restraint when the actual column stiffness may be reduced due to yielding caused by residual strains in addition to the strains caused by the applied load. Yura developed an expression that allows for inelastic column behavior in the calculation of the constant, G, (Equation 4.6) that is used in Equations 3.25 and 3.26 to calculate the K for a restrained column.

In Yura's proposal, an elastic value of G is calculated using Equation 4.6. This value is then modified by

$$G_{inelastic} = \frac{F_a}{F_e'} G_{elastic} \tag{4.10}$$

where F_a is the allowable stress for a column subject to axial load only

F_e' is the allowable stress in a column using Euler's equation and a factor of safety of 1.92.

In the case of an elastic column Fa=Fe' and there is no modification of G. Yura suggested that in the case of an inelastic column, Equation 4.10 should be applied iteratively until G has converged to the smallest possible value (7). For each iteration, the values Fe' and Fa are based on the K calculated in the previous cycle. Yura suggested that K would converge to 1 for most inelastic columns.

In the original proposal, Yura only considered the use of the inelastic K factor for the case of axially loaded members. Adams (33), however, questioned the use of such a K factor in the interaction equation for beam columns (Equation 3.21). In his reply, Yura, suggested that an inelastic K factor should only be applied when calculating the allowable stress when only axial load is present and should not be applied to the moment magnification (34).

To study the implications of Yura's proposed K factor for the design of beam columns, K factors calculated using his method were used in Equation 3.21 and the resulting interaction curve was compared with the prediction of the ultimate strength analysis outlined in Section 3.2. The results of these analyses are shown in Figures 4.8 and 4.9 for h/r's of 20, 40, and 60, and Gelastic of 0.5 and 4.0. In all cases, σy = 36 ksi. It must be noted that Yura's proposal was related to an allowable stress specification, while in this study, all analyses have been performed using a Limit States Design procedure (24). Thus, Equation 4.10 is modified to

$$\text{Ginelastic} = \frac{C_r}{C_e}\,\text{Gelastic} \qquad (4.11)$$

where C_r is the factored compressive resistence of a member for axial

load, and

C_e is the elastic buckling load.

The differences between the results of Equations 4.10 and 4.11 were found to be small (less than 5%).

While the K factors calculated using Yura's suggestion were reduced from the K factors calculated from an elastic model, in no case, did the K factor actually reduce to 1 as Yura suggested (7).

As seen in Figure 4.8 and 4.9, the interaction predicted by Equation 3.21 using an inelastic K factor (shown as the dashed line) is quite close to the interaction predicted by the ultimate strength analysis (shown as the solid lines) for h/r = 20 for G = 0.5 and G = 4.0. As the column slenderness (h/r) increases, however, the interaction predicted using the inelastic K factor becomes conservative when compared to the ultimate strength prediction.

Thus, while Yura's proposal does provide an excellent prediction for low values of column slenderness (when inelastic action in columns is more prevalent), the procedure does not provide a more consistent margin of safety for all values of column slenderness than K factors based on an elastic model. Indeed, as a practical design method, the proposed method has some disadvantages. The procedure can involve lengthy calculations that may not result in significant material savings. In addition, the designer may be confused by the use of two different K factors in the same equation.

In his discussion of the original paper, Adams questioned the consideration of inelastic column behavior while ignoring the reduction of beam stiffness due to inelastic action. Experimental studies

have shown that there is little reduction to column strength due to beam yielding unless the beam has become fully plastic (44, 45). To date, however, there has been no experimental or analytical confirmation of the suggestion that inelastic column behavior will increase the effective end restraint on the column.

The results of the analyses performed in this section does show that the use of K factors based on inelastic column action does provide a reasonable prediction of beam column behavior, particularly at low slenderness ratios. It is important to note that a large percentage of columns used in medium height buildings will be found in this slenderness range.

Subject to the practical limitations discussed, Yura's proposal does appear to have merit when applied to a limit states design approach to design of building frames. This method, however, does not appear to be a further rationalization of design for stability, only a method whose results are a reasonable prediction of member behavior for a range of member sizes.

4.5 FRAMES WITH MEMBERS OF VARYING STIFFNESSES

As outlined in Section 4.2, a major problem in the design of beam columns is relating the overall stability of the structure or a critical portion of the structure to the design of a particular member. This problem is particularly difficult when a column with ends effectively pinned is supported by a column with rigid end restraints. In the traditional procedures, a pin ended column that is not supported by a bracing system (sway-permitted) is considered to have an effective

length that approaches infinity. Thus, the allowable axial load for the member is zero.

In addition to his suggested method for calculating K on the basis of inelastic column stiffness, Yura also proposed methods for evaluating the overall capacity of a frame in which the columns vary in stiffness (7, 34). In the initial paper, Yura suggested that a storey of a plane frame should be designed to satisfy the criteria that the sum of the vertical forces applied to the members of the frame shall be less than or equal to the sum of the resistance to vertical load of all members of the storey. If a column is effectively pin ended, the resistance of that column to vertical forces in the sway condition is zero. To ensure that the frame does not fail by the buckling of one member, each member is checked for its ability to carry its own load with K based on the sway prevented condition.

In his later discussion of the paper, Yura extended this concept to design of beam columns by suggesting the moment magnifier, β, (an ultimate strength form) be replaced with

$$\beta = \frac{1}{1 - \Sigma P/\Sigma P'_e} \qquad (4.12)$$

in the stability check (Equation 3.21) for each member where

ΣP = sum of the axial forces in the storey in question,

$\Sigma P'_e$ = sum of the elastic buckling resistances for each member in the storey (note for $K \Rightarrow \infty$, $P'_e \Rightarrow 0$).

The effect of this proposal is that the form of moment magnification is related to the overall stability of the storey. To study this proposal, the analysis outlined in Section 3.2 was modified to con-

sider a beam column subassemblage linked to pin ended column. This was accomplished by multiplying $P\,\Delta/h$ in the shear equilibrium equation by an integer factor, n. The interaction as predicted by the stability equation was found by the following:

$$\frac{C_f}{C_r} + \frac{\omega M_f}{M_r\left(1 - \frac{\Sigma C_f}{\Sigma C_e}\right)} \leq 1.0 \qquad (4.13a)$$

where ΣC_f is nC_f and ΣC_e is C_e for the supporting column.

The comparison of these interaction curves for various values of h/r, G, and n are shown in Figures 4.10, 4.11, and 4.12. In each case, the theoretical ultimate strength interaction is shown as the solid line and the corresponding prediction, using Equation 4.13a with K based on the traditional model is shown as the dashed line. For the examples shown, although the factor of safety is not consistent for all values of h/r and G, Equation 4.13a provides a reasonable prediction of member ultimate strength.

More recently, however, Kanchanalai, has presented a modification to Yura's proposal (49). In limit states form, Equation 3.13a would be modified as (49):

$$\frac{C_f}{C_r} + \frac{M_f}{M_r\left(1 - \frac{\Sigma C_f}{\Sigma C_e}\right)} \leq 1.0 \qquad (4.13b)$$

The object of this equation would be to check member strength.

In addition, the stability of each storey would be checked by (49):

$$\frac{\Sigma C_f}{\Sigma C_r} \leq 1.0 \qquad (4.14)$$

In his study of the design of beam-columns, Kanchanalai, showed that the moment magnifier in Equation 4.12 provided a somewhat conservative prediction of second order moments in rigid frames. Kanchanalai also compared the predicted moment-axial load interaction provided by Equation 4.13b with the results of an ultimate strength analysis for single storey rigid frames. While the resulting curves showed that Equation 4.13b could provide a reasonable prediction of member behavior, the single storey model used is not well related to actual member behavior in multi-storey frames.

The prediction provided by Equation 4.13b of the interaction between moment and axial load for the model outlined in Section 3.2 is shown as the broken line in Figures 4.10, 4.11, and 4.12. In each case, the prediction is quite conservative when compared with the ultimate strength analysis. Thus, while the use of the moment magnifier as suggested by Yura, does provide a method for considering "leaned frames" using traditional effective length procedures, the modification presented by Kanchanalai is not justified.

While Kanchanalai performed experiments in an attempt to confirm the validity of Equation 4.14, he did not consider the effect of this limit on the "leaned frame" system. The limit imposed by this equation can be considered for a given h/r, G, σ_y and n by the expression:

$$\frac{P}{P_y} = \frac{C_r}{A\sigma_y n} \tag{4.15}$$

Thus, for h/r = 20, G = 0.5, σ_y = 36 and n = 4, the value of P/P_y would be limited to 0.22. As shown in Figure 4.10, this limit is not related to the actual ultimate strength of the subassemblage.

The use of this Equation 4.14 for a mixed framing system could result in the design of the more rigid members of the system to carry the vertical loads applied to all members of the system. In reality, each member should be designed to carry its design axial load, while the more rigid members may be required to carry the second order (or sway) effects of the other members in the system. Thus, Equation 4.14 does not necessarily reflect the behavior of the mixed framing system.

4.6 SUMMARY

The thrust of most proposals to modify the design of structures for stability has been some modification of the effective length factors used in the design of axially loaded members. The rational used is that the model traditionally used for the calculation of effective lengths does not reflect true structural behavior. The main problem is to relate the overall stability of a structure to the design of a specific member.

Unfortunately, the proposed modifications have only dealt with the calculations of the effective length factors and not how or why these factors are applied to design formulae. Since the use of

effective length factors in design formulae reflects an essentially empirical method of accounting for neglected second order effects, a more rational model for the calculation of the effective length does not necessarily result in a more rational accounting for second order effects.

In this chapter, the results predicted when a proposed effective length factor was used in the stability interaction equation (Equation 3.21) were compared to the predicted member response obtained for an ultimate strength analysis. This comparison is a measure of how the proposed modification actually predicts member strength when used in design formulae.

None of the proposed methods provided a consistent prediction of member behavior. In some cases, the predictions provided were unrealistically conservative. In other cases, the prediction of member strength for combined axial load and bending was unconservative. In no case was there an actual assessment of overall structural stability under combined loading. A truly rational design method should adequately predict member behavior and provide an assessment of the overall stability of the structure.

TABLE 4.1

EFFECTIVE LENGTH FACTORS FOR TEN STORY FRAME

Column	Story	K_T	K_F	K_S	λ_C
1	1	1.43	1.45	1.35	15.20
2	1	1.31	1.24	1.15	
3	1	1.34	1.27	1.17	
4	1	1.52	1.53	1.42	
1	2	2.01	1.93	1.93	13.04
2	2	1.67	1.63	1.63	
3	2	1.75	1.67	1.67	
4	2	2.27	2.02	2.02	
1	3	2.03	1.87	1.71	15.50
2	3	1.72	1.64	1.50	
3	3	1.80	1.61	1.48	
4	3	2.30	1.87	1.72	
1	4	2.12	2.00	1.91	14.39
2	4	1.82	1.75	1.67	
3	4	1.88	1.72	1.64	
4	4	2.40	2.01	1.92	
1	5	2.11	1.85	1.75	14.48
2	5	1.82	1.63	1.54	
3	5	1.82	1.47	1.40	
4	5	2.32	1.73	1.65	
1	6	1.97	2.04	1.87	15.54
2	6	1.69	1.79	1.64	
3	6	1.67	1.62	1.48	
4	6	2.11	1.92	1.75	

TABLE 4.1 (cont'd)
EFFECTIVE LENGTH FACTORS FOR TEN STORY FRAME

Column	Story	K_T	K_F	K_S	λ_C
1	7	1.88	1.90	1.69	16.51
2	7	1.58	1.50	1.33	
3	7	1.58	1.37	1.22	
4	7	2.00	1.69	1.50	
1	8	1.80	2.24	1.91	17.92
2	8	1.47	1.75	1.49	
3	8	1.45	1.61	1.37	
4	8	1.91	1.99	1.70	
1	9	1.68	2.28	1.77	21.76
2	9	1.34	1.52	1.18	
3	9	1.24	0.91	0.70	
4	9	1.76	1.98	1.53	
1	10	1.62	3.85	2.63	27.98
2	10	1.25	2.34	1.60	
3	10	1.10	1.39	0.95	
4	10	1.56	3.40	2.32	

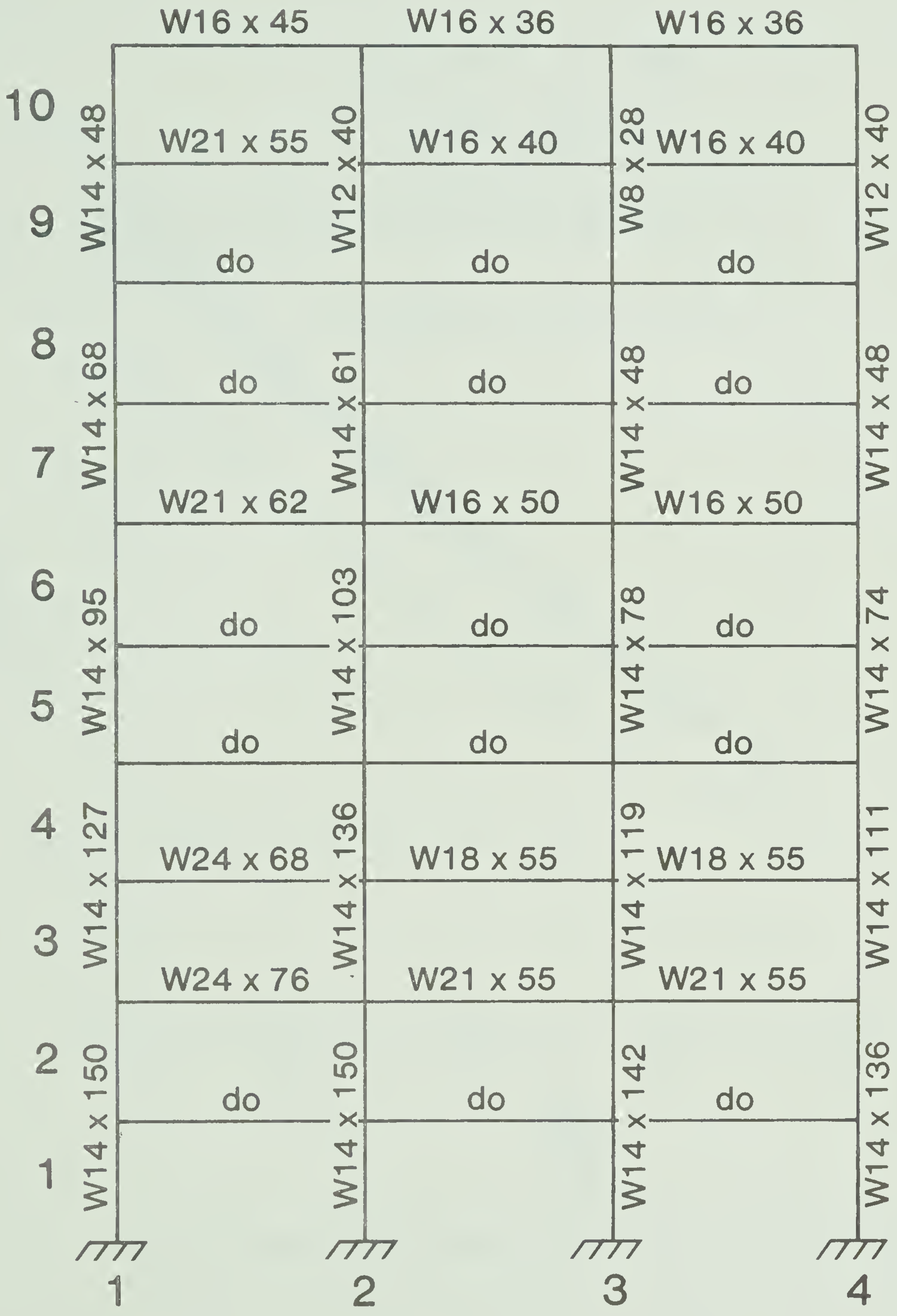


Figure 4.1 Lehigh Ten Storey Frame Elastic Design

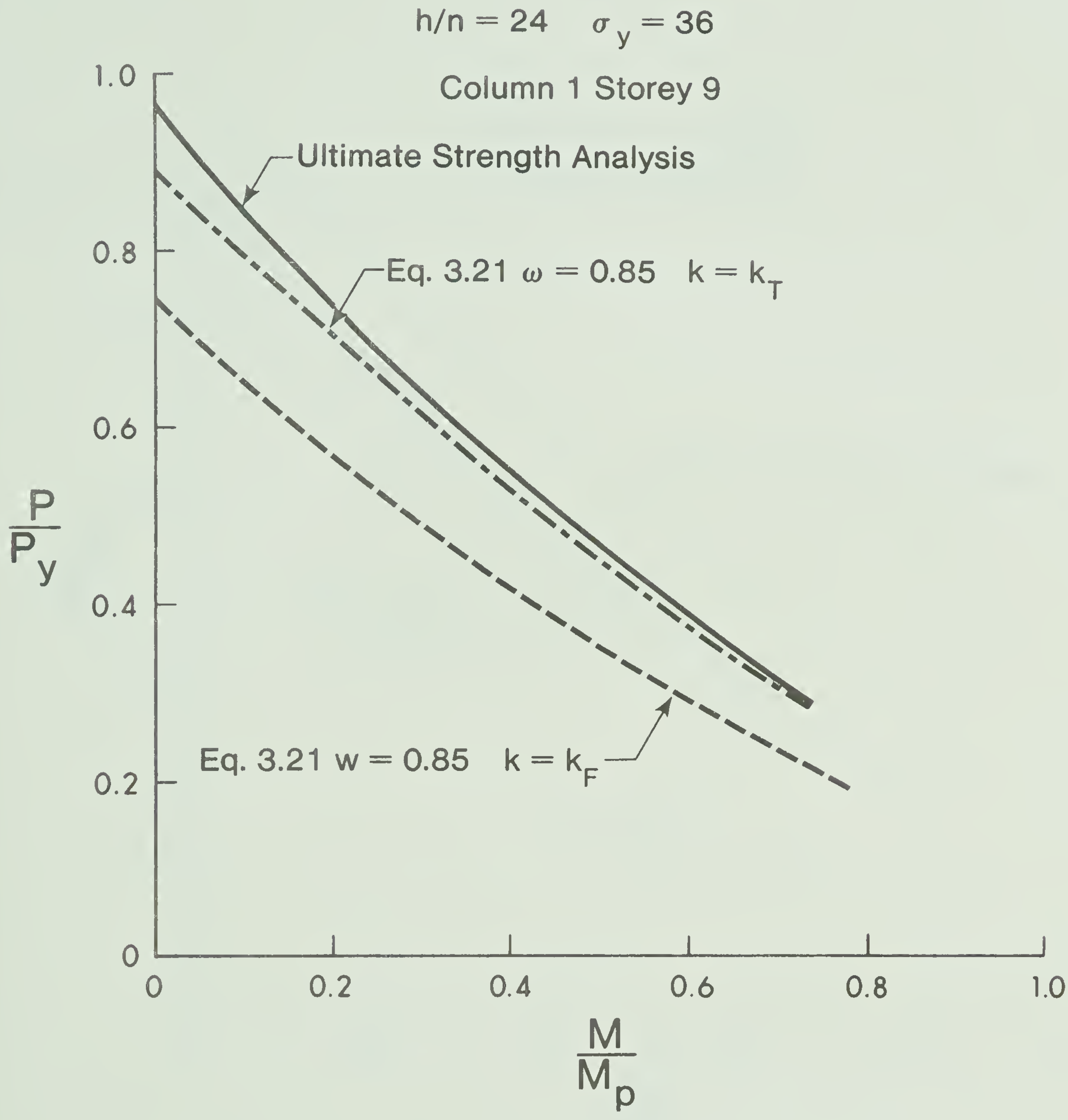


Figure 4.2 Interaction Prediction Using Frame Buckling K Factor

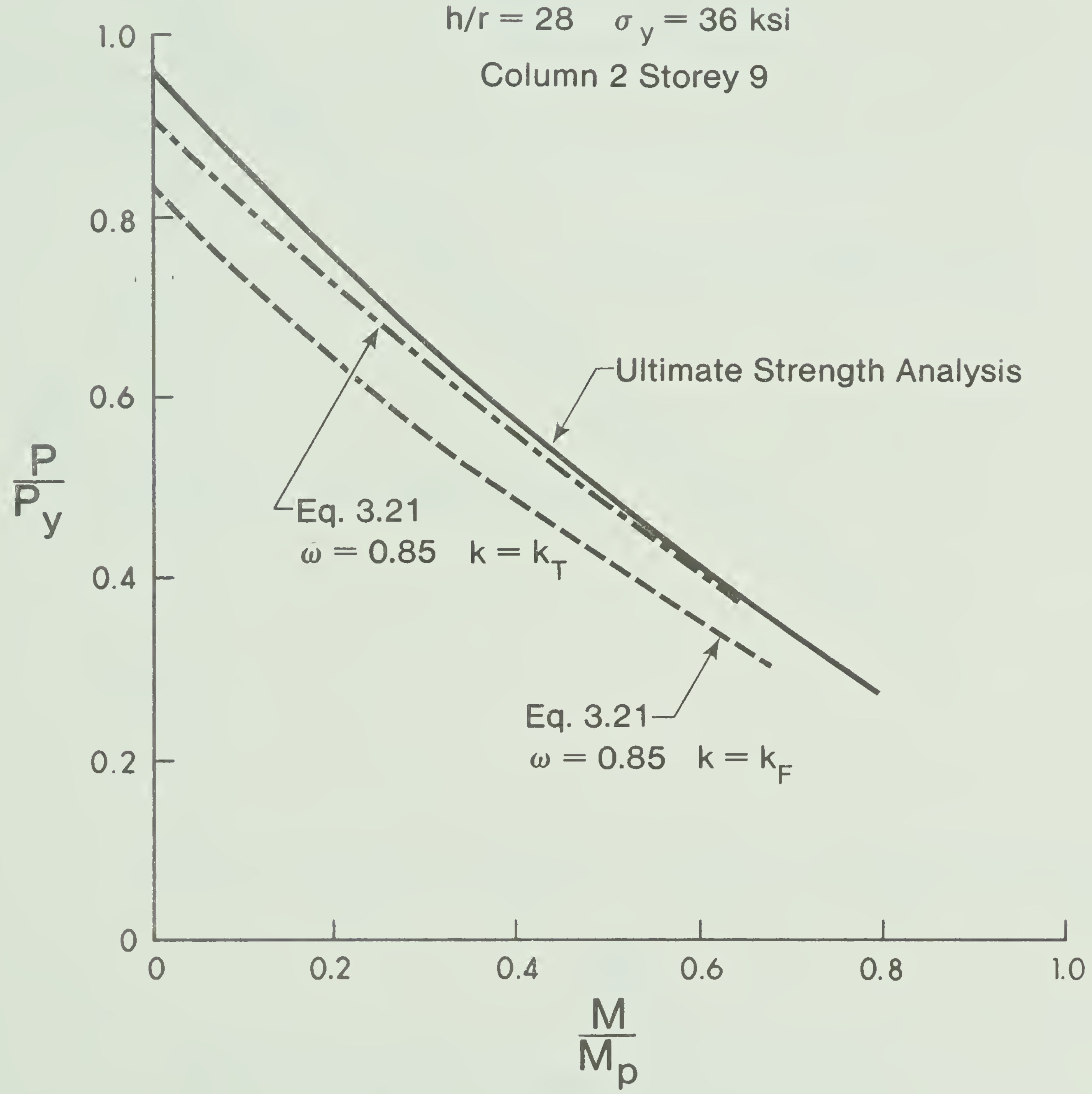


Figure 4.3 Interaction Prediction Using Frame Buckling k Factor

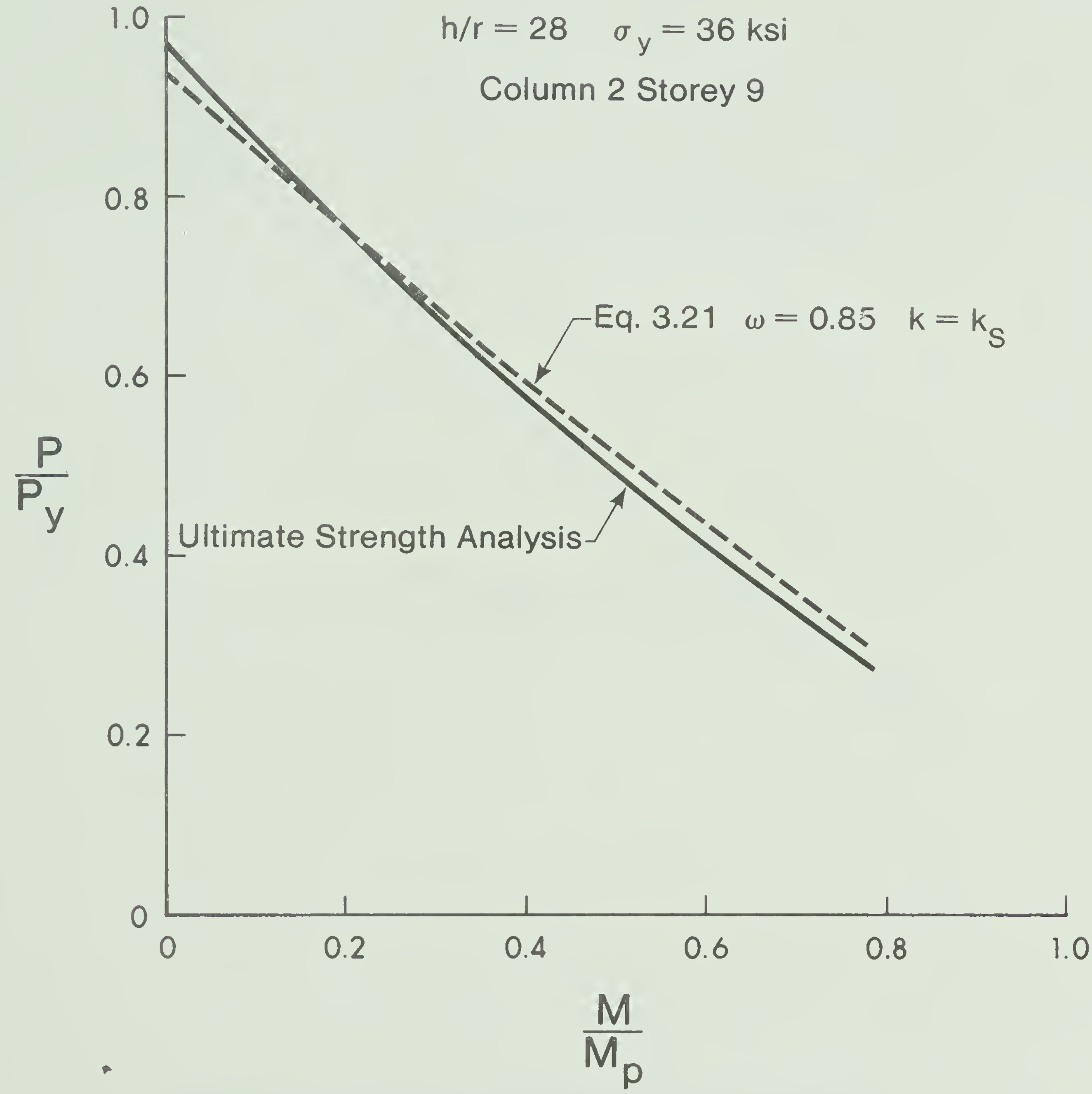


Figure 4.4 Interaction Prediction For Storey Buckling K

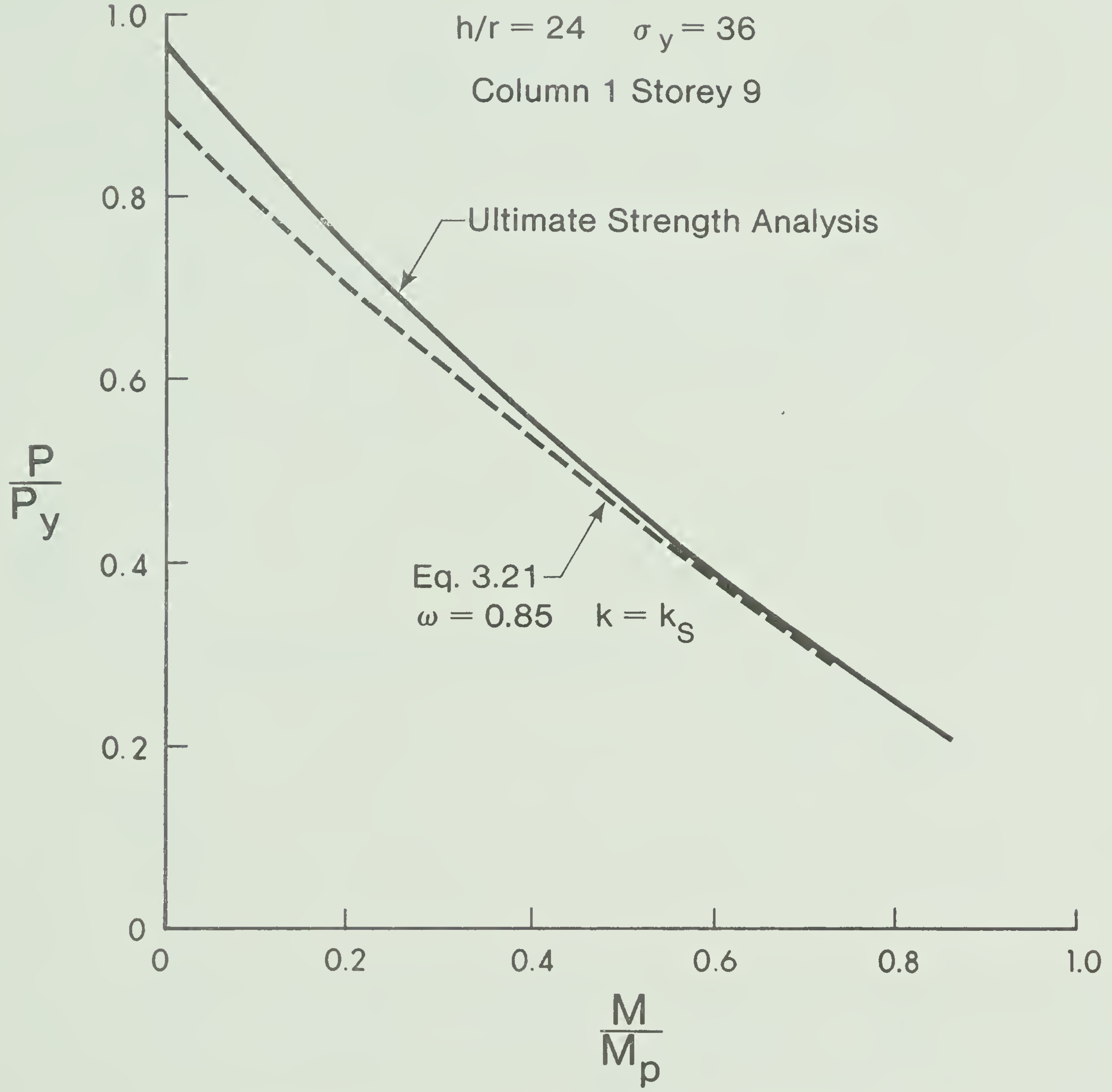


Figure 4.5 Interaction Prediction For Storey Buckling K

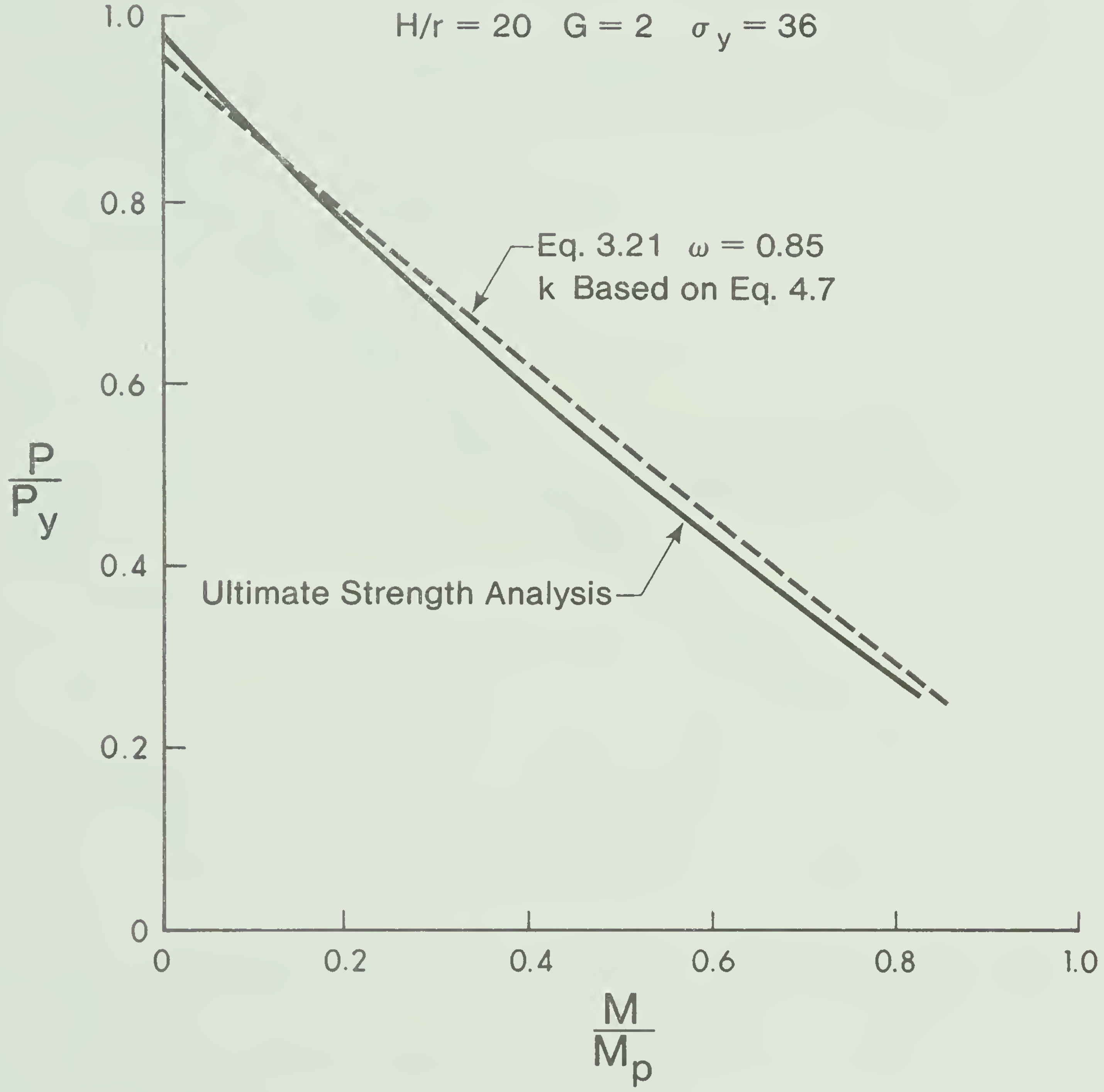


Figure 4.6 Interaction Prediction Using Lay's Modified K Factor

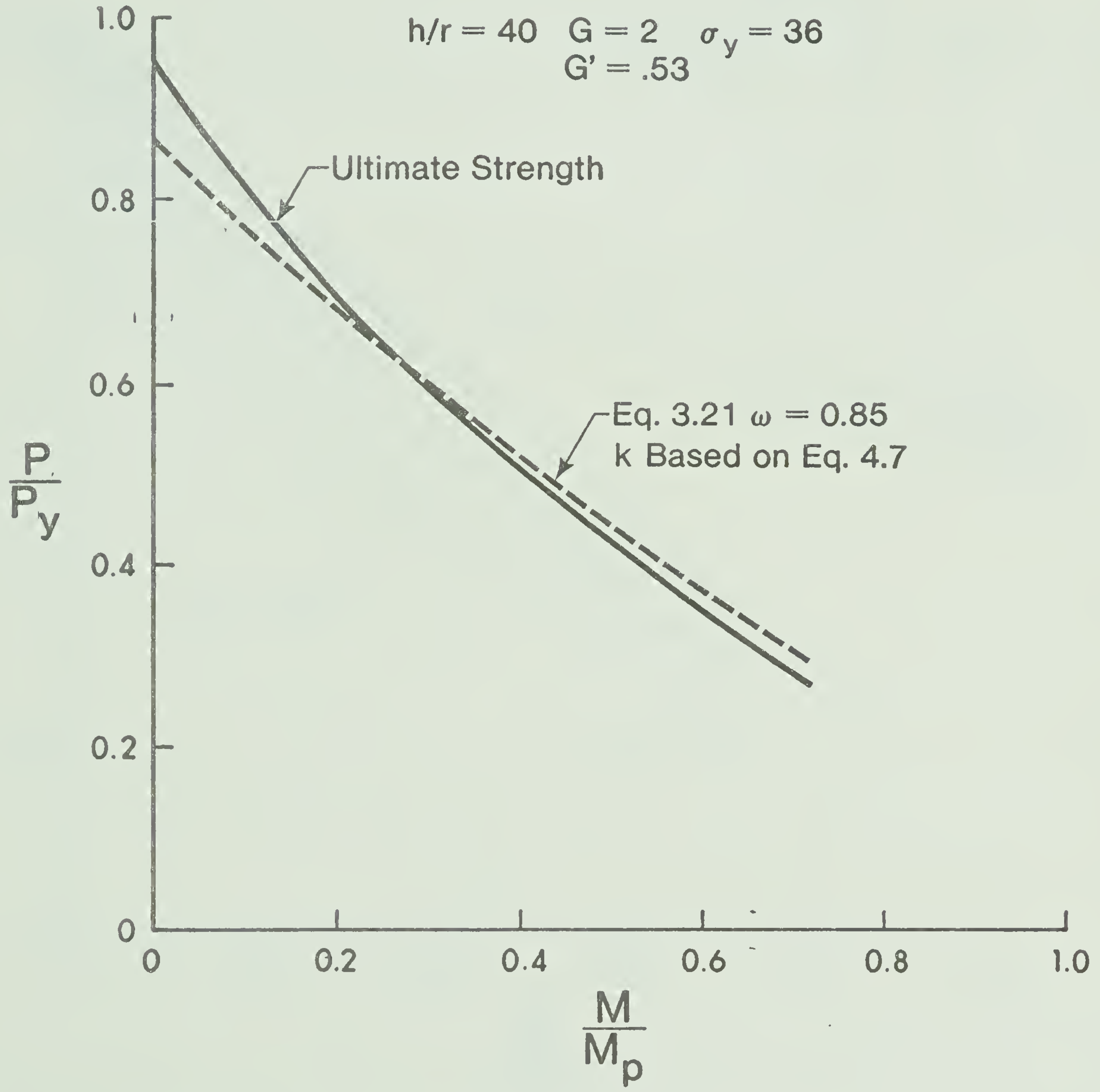


Fig. 4.7 Interaction Prediction Using Lay's Modification Factor

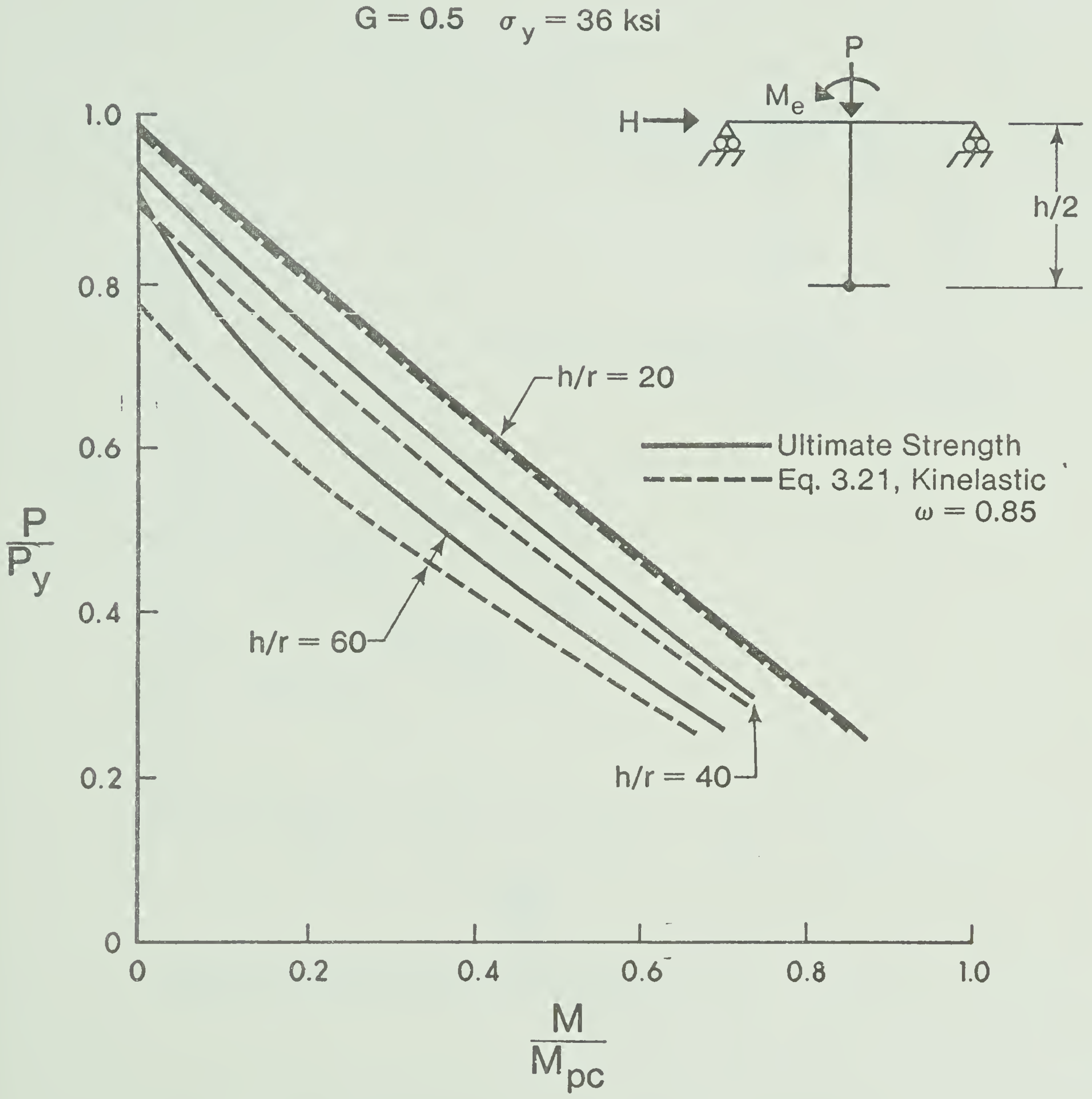


Figure 4.8 Interaction Prediction Using Inelastic K Factors

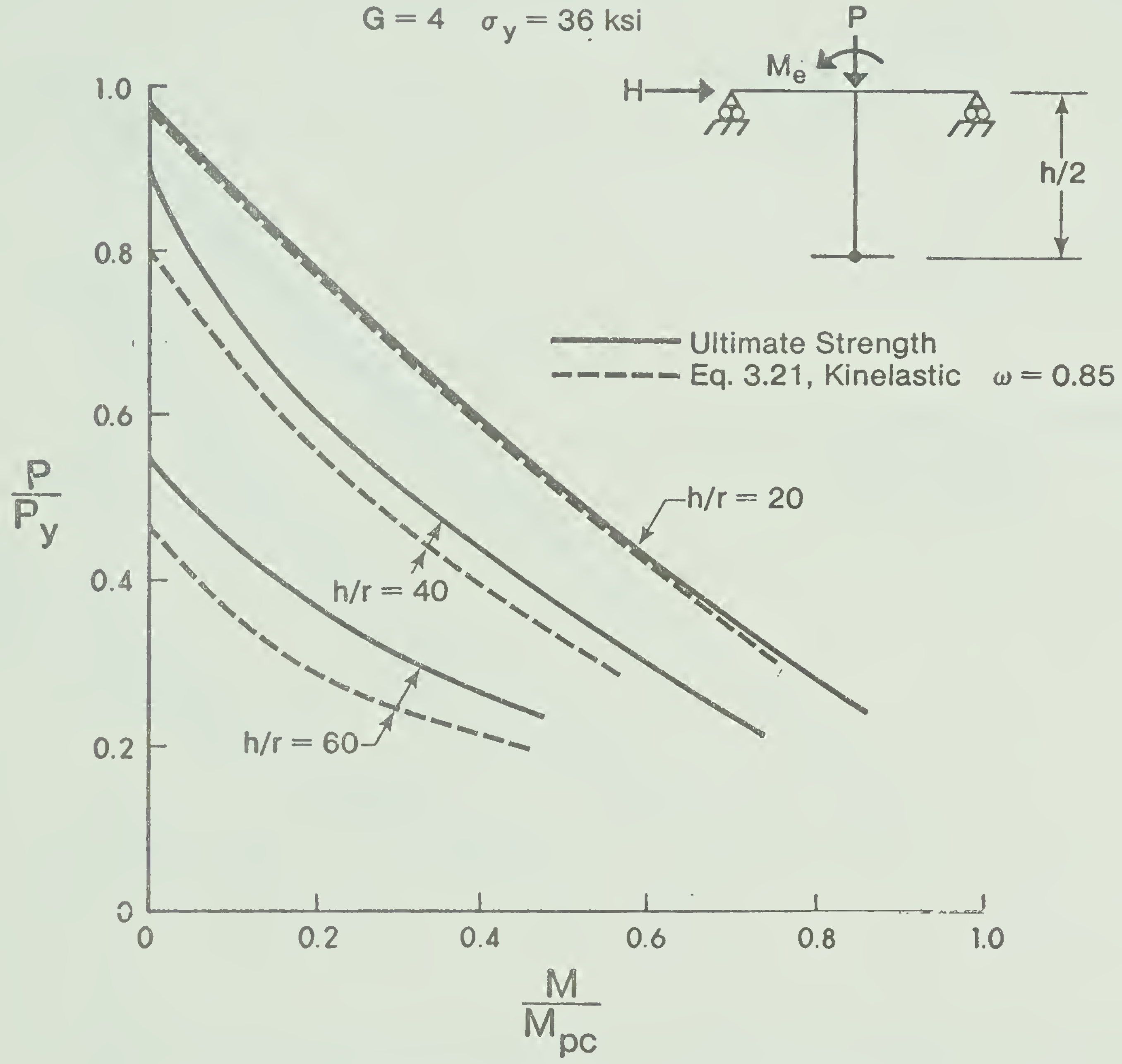


Figure 4.9 Interaction Predictions Using inelastic K

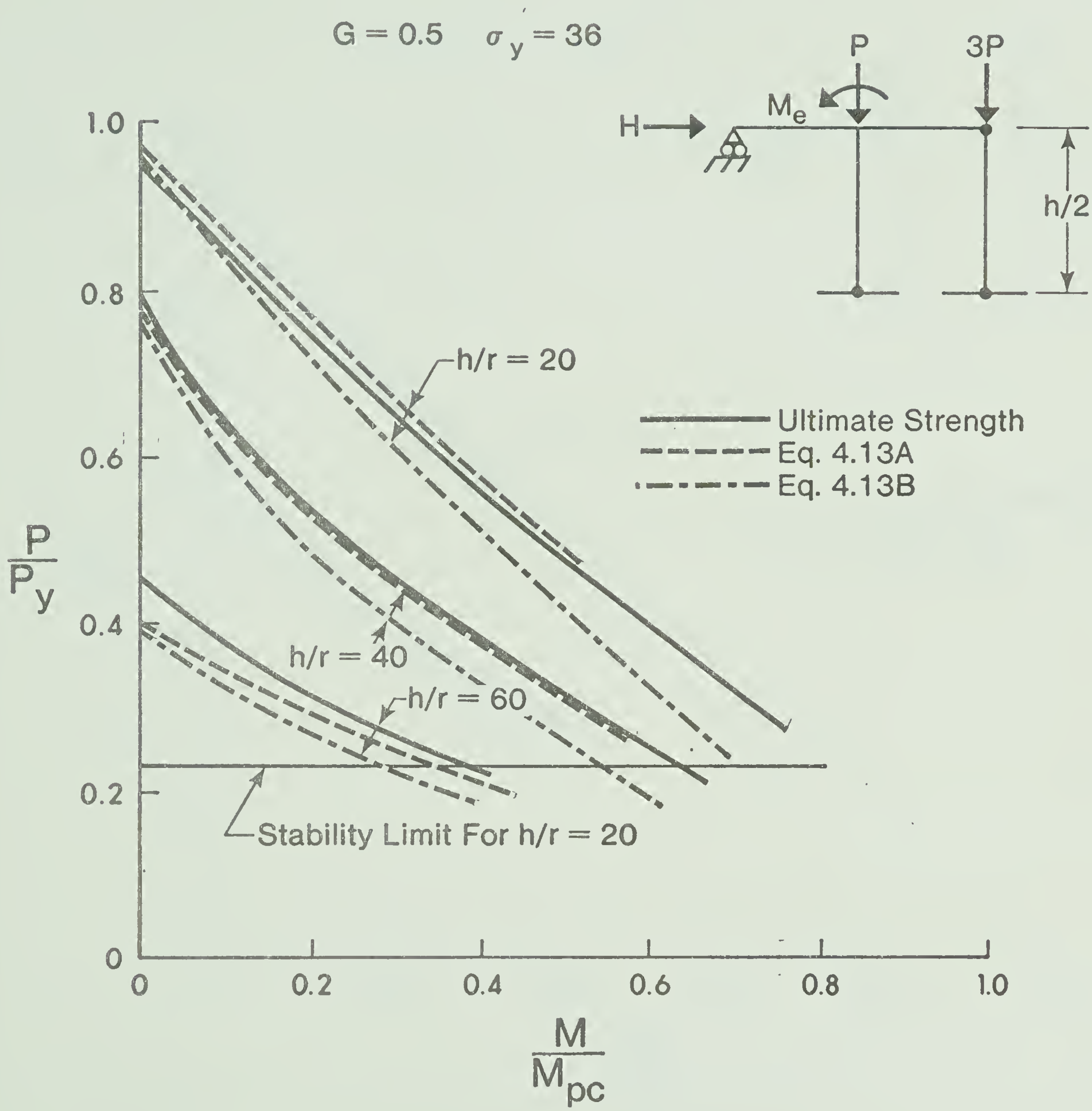


Figure 4.10 Interaction Predictions Using Equation 4.13

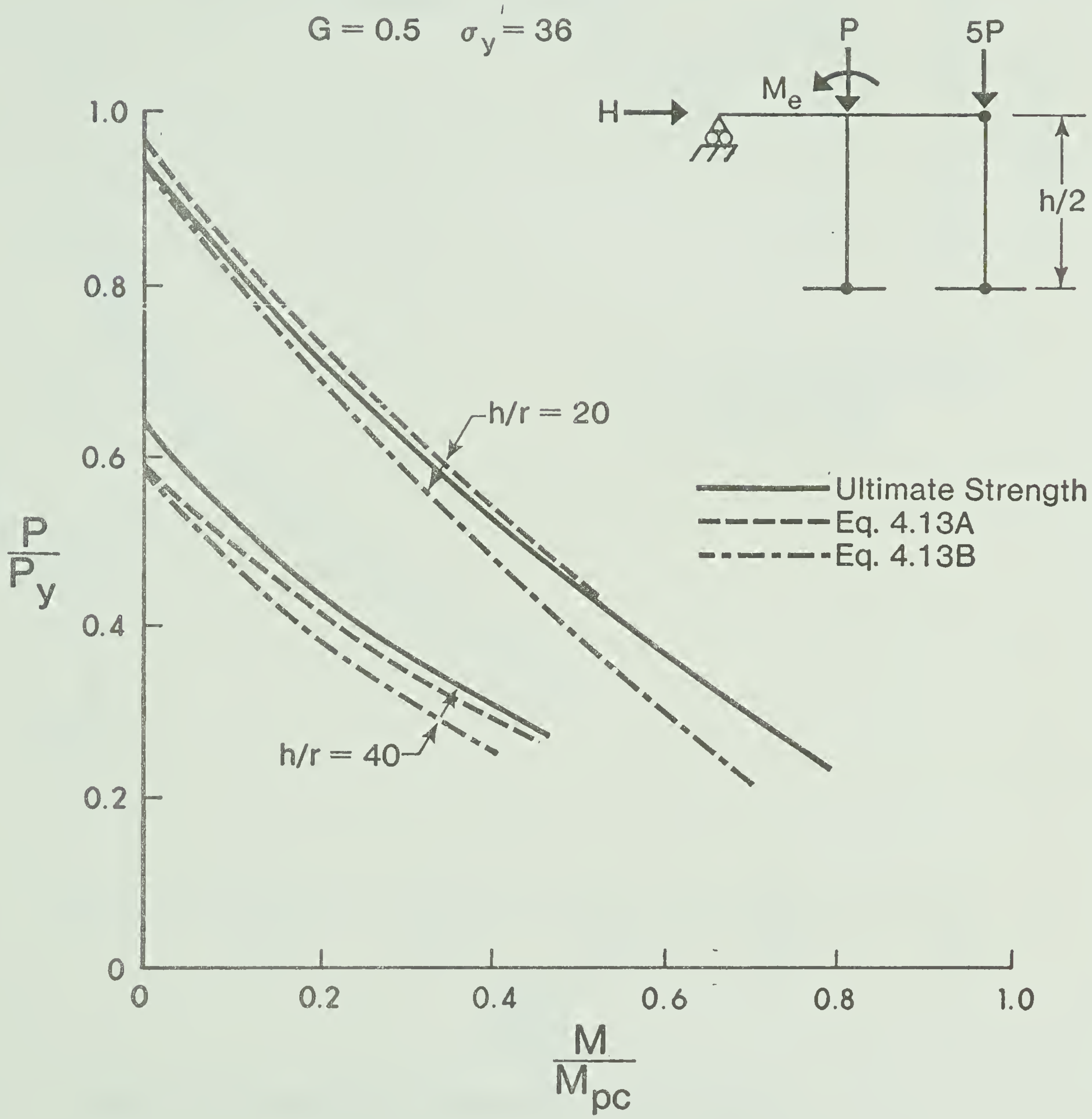


Figure 4.11 Interaction Predictions Using Equation 4.13

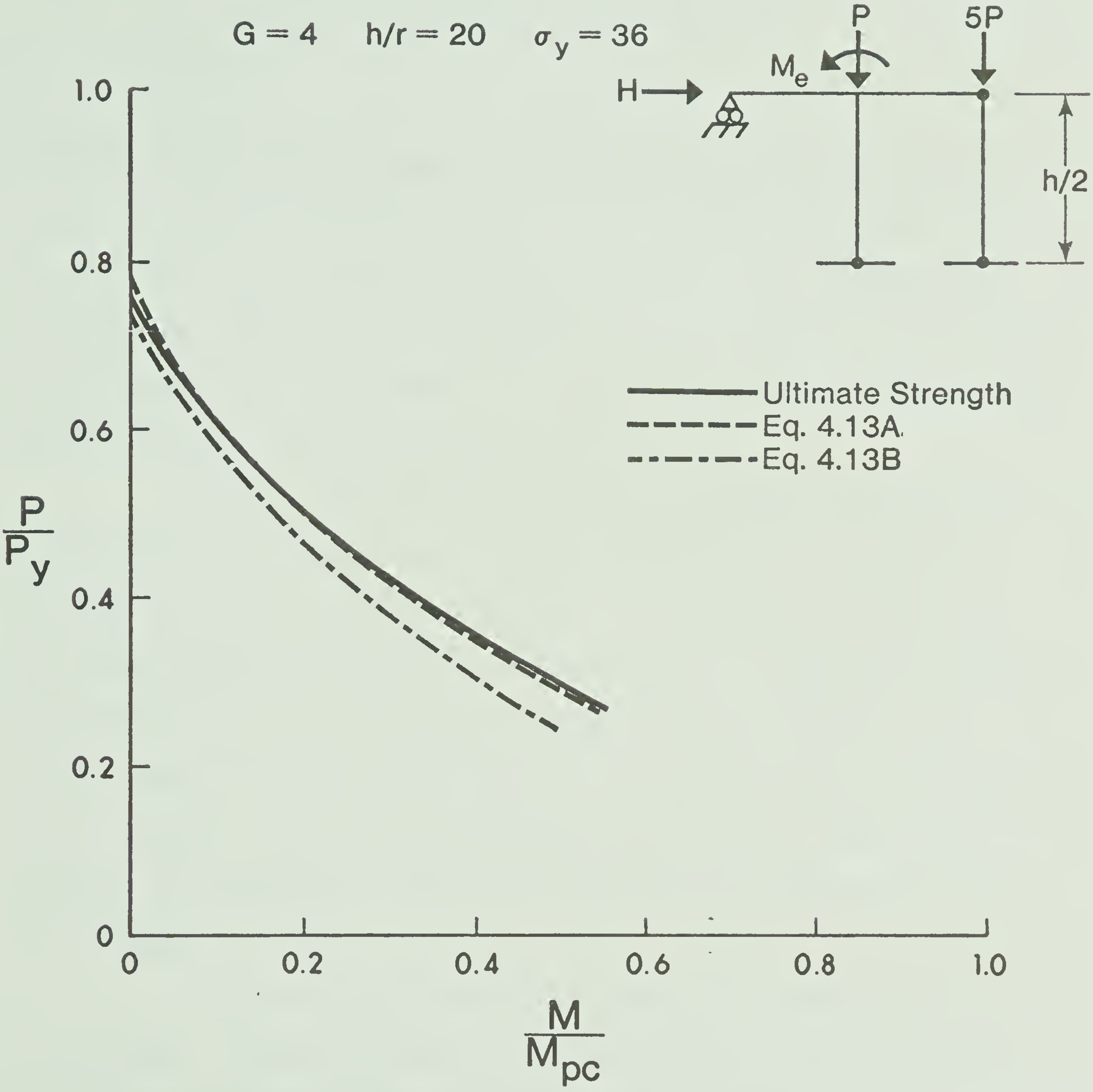


Figure 4.12 Interaction Predictions Using Equation 4.13

CHAPTER V

COLUMN DESIGN BY THE P DELTA PROCEDURE

5.1 INTRODUCTION

The analyses used to determine the distribution of bending moments and internal forces throughout the structure are generally first order. This type of analysis neglects the effect of sway deformations on the shear equilibrium equations, and also, the influence of axial force on the stiffness of the members (1). A first order analysis overestimates both the stiffness and the strength of a frame (1).

If the axial forces are significant, a better indication of the distribution of the interal moments and forces in the structure is obtained by using a second order analysis. In a second order analysis, the shear equilibrium equations are formulated on the deformed structure. In other words, the secondary moments produced by the vertical loads acting through the sway displacements of the structure are accounted for in the analysis. These secondary moments and forces are termed the PΔ effects. The influence of axial force on the column stiffness may also be considered.

In this Chapter, the P Delta Procedure of column design will be outlined. Factors that can effect design using the P Delta Procedure are discussed as well as limitations that may be required. Factors to be discussed include, inelastic beam action, effect of axial load on column stiffness, frame flexibility, and other pertinent behavior.

5.2 P DELTA DESIGN PROCEDURE

The difficulties with the present design procedure arise from the fact that the neglect of the PΔ forces in the analysis of the structure is only imperfectly compensated for in the selection procedure for individual members. These difficulties could be overcome by performing analyses which do include the second order effects. Standard computer programs are available to perform a second order analysis, but the use of these programs for the analysis of a large frame are expensive (14,42). The results of a standard first order analysis may be easily modified to include the PΔ effect.

The initial step in the modification procedure is to compute the sway forces. The lateral and vertical loads are first applied to the system, and the lateral displacements, denoted as Δ_i in Figure 5.1, are computed by first order theory. The floor level is denoted by i. The additional storey shears due to the vertical loads are then computed:

$$V_i' = \frac{\Sigma P_i}{h_i} (\Delta_{i+1} - \Delta_i) \tag{5.1}$$

where V_i' = additional shear in storey i due to the sway forces,

ΣP_i = sum of the column axial loads in storey i.

h_i = height of storey i; and,

Δ_{i+1}, Δ_i = displacements of level i + 1 and i, respectively.

The sway forces due to the vertical loads, H_i', are then computed as the difference between the additional storey shears at each level:

$$H_i' = V_{i-1}' - V_i' \tag{5.2}$$

The sway forces, H_i', are added to the applied lateral loads, and the structure re-analyzed. When the Δ_i values at the end of a cycle are nearly equal to those of the previous cycle, the method has converged, and the resulting forces or moments in every member now include the PΔ effect.

The method described above is applicable to both the combined loading case and to the vertical load only case. In both cases, the deflected shape under the applied loads is determined and the sway forces calculated using Equations (5.1) and (5.2). The sway forces are then applied to the structure and the deflected shape again determined. In some instances, for the vertical load only case, the initial application of the vertical loads will not produce a significant sway displacement of the storey. In these cases, initial sway forces may be computed on the basis of a storey rotation equal to the probable erection tolerance.

Once the process has converged, the moments and forces include an allowance for the PΔ effect. In most cases for practical structures, the convergence is extremely fast and the first iteration produces acceptable results (11).

Since the PΔ moments have now been included in the analysis, they need not be accounted for in the design. Thus, the design of the beam-columns may be based on Equations 3.19, 3.20, and 3.21, but with the computation of K based on the sway prevented model and ω given by Equation 3.22. This procedure requires that the columns and girders be designed for increased moments to resist the PΔ effect.

The procedure suggested is basically equivalent to designing

on the envelope provided by the upper dashed lines (Equation 3.21 with K based on the sway prevented model and ω given by Equation 3.22) and the broken line (Equation 3.20) in Figure (5.2). In effect, the total moment capacity of the member is now to be matched against the total (primary and $P\Delta$) bending moment applied to the member, as shown by the upper solid line in Figure (5.2). For the condition shown in Figure (5.2), the interaction equations provide a close estimate of the ultimate capacity of the member.

The designer would simply analyze the structure to be considered under the various load combinations considered significant. In each case, the sway forces would be computed and added to the applied lateral loads on the structure. The influence of these additional loads would be evaluated and, if necessary, a second iteration could be performed in the analysis. The normal limit state would be used in the girder design, while the columns would be evaluated using the interaction equations with K based on the sway prevented model and ω evaluated as for the sway prevented case (1).

When using an allowable stress design, however, the use of equations 5.1 and 5.2 to obtain sway forces may result in an underestimation of moments at point of ultimate capacity in the critical member. Thus, Equation 5.1 must be modified for use in allowable stress design. This modification is presented in Appendix A.

5.3 EFFECT OF INELASTIC ACTION

In the $P\Delta$ technique described above, the bending moment and force distributions are computed on the basis of an elastic analysis of

the structure, which includes the PΔ effect. The various elements in the structure will accept the moments and forces caused by the PΔ effect in proportion to their stiffnesses. Since the PΔ effects are known and are included in the analysis, the designer need not provide a reserve of strength in selecting the actual member.

The limit states design technique generally implies that a safety index is maintained against the attainment of the ultimate capacity of a critical member. Except for limited recognition of moment redistribution, the attainment of the ultimate capacity at one location in the critical member is assumed to correspond to the ultimate strength of the structure.

At this stage of loading, however, some inelastic deformation will have occurred in various parts of the structure. For example, using published moment-rotation relationships for beams under moment-gradient (27,37), the rotations corresponding to the attainment of the ultimate moment capacity, M_p, are approximately 1.3 times those which would be computed on the basis of elastic action. Similar results have been obtained from tests intended to simulate other structural configurations (27). Although only one or a few members may yield before the first plastic hinge has formed, these limited inelastic deformations will tend to increase the PΔ effect above that computed on the basis of elastic action.

In addition to the inelastic action described above, columns subjected to high axial loads may yield along their lengths due to the strains caused by the axial loads. This situation will occur when the axial load is greater than 0.7 times the yield load, assuming that the

maximum residual stress is 0.3 F_y (27).

Theoretically, yielding of the flange tips will result in a reduced stiffness for the column and increased deflections (and thus, $P\Delta$ effects) above those computed elastically. In most cases, however, the axial column loads (at the load level corresponding to the attainment of the ultimate capacity of the critical member) are below the level required to cause significant yielding. Where the members are subjected to high axial loads, the sway deflections will be increased above those computed elastically, only for that portion of the loading history for which the axial loads exceed 0.7 times the yield load.

Theoretically then, the $P\Delta$ forces should be based on deflections that exceed those calculated on the basis of elastic behavior. Because of the conservative nature of the column design procedure (1) and because the test results on large scale specimens have not revealed significant inelastic deflections at the loads corresponding to the formation of the first plastic hinge (27), the $P\Delta$ technique ignores inelastic deflections. The influence of member yielding will be considered at length in the following chapter.

5.4 EFFECT OF FRAME FLEXIBILITY

As outlined in section 5.2, the $P\Delta$ Procedure implies that the member capacity appropriate to the sway prevented condition is available to resist computed moments and forces in the member. In a braced structure, if the bracing system is designed to resist both applied lateral forces and sway forces, the full capacity of the member is available to resist the applied moments and forces. For unbraced

structures, this may not always be true.

Consider the column shown in Figure 5.3(a) subjected to an axial load P and a transverse load H. The column is pinned at the base and restrained by a rotational spring at the top. The spring represents the restraining action of the girders as the column is permitted to sway. In the deformed position shown in Figure 5.3(b), the column end moment, M_C, must be equal to:

$$M_C = Hh + P\Delta \tag{5.3}$$

and must be equal and opposite to the girder (spring) end moment. For compatibility, the end rotation of the column, γ, (from its chord) and the girder end rotation, θ, are related by:

$$\Delta/h = \gamma + \theta \tag{5.4}$$

The results of an analysis of the system are shown in Figure 5.3(c), where the moment capacities used up by the various forces are plotted against the deflection, Δ. The $P\Delta$ term is shown as the dashed line, while the resisting moment developed at the column top ($M_C = Hh + P\Delta$) is shown as the upper solid curve. This curve reaches a maximum value of M_{pc}, the reduced plastic moment capacity of the member. The lower solid curve plots the portion of the resisting moment used by the first order moment (Hh). From Equation 5.3, this curve reaches a maximum where the rate of change of the resisting moment is equal to the $P\Delta$ effect:

$$\frac{\partial (Hh)}{\partial \Delta} = \frac{\partial M_C}{\partial \Delta} - P \tag{5.5}$$

The deformation at this stage is lower than that corresponding to the attainment of the maximum moment capacity M_{pc}. Thus, even though

the $P\Delta$ moments are included in the analysis (in the $P\Delta$ method), the full moment capacity, M_{pc}, is still not available to resist the total calculated moments.

To illustrate this point, the results of several analyses of beam columns free to sway are shown in Figures 5.2, 5.4, and 5.5. The column is shown in the insert to each figure and the results correspond to the attainment of the ultimate load capacity. The resisting moment at the top of the column, M_c, is non-dimensional and M_C/M_P and plotted against the axial load ratio, P/P_y, as shown by the lower solid curve in the figures. The portion used by primary moment, $M_C = Hh/2 + P\Delta$ is shown as the upper solid curve. For a given P/P_y value, the difference between the two curves is a measure of the moment required to resist the $P\Delta$ effect.

In these figures, the upper dashed curve representes the results of the stability interaction equation (24).

$$\frac{C_f}{C_r} + \frac{\omega M_f}{M_r(1 - \frac{C_f}{C_e})} \leq 1.0 \tag{5.6}$$

based on the sway prevented model. The lower dashed curve represents Equation 5.6 based on the sway permitted model. The broken curve represents the strength equation (24,43).

$$\frac{C_f}{C_r} + \frac{0.85M_F}{M_r} \leq 1.0 \tag{5.7}$$

The envelope provided by Equation 5.6 with w and K based on the sway prevented condition and Equation 5.7 has been found to provide an adequate prediction of the ultimate strength of beam columns whose

ends are prevented from translation (31). When a structure contains a stiff vertical bracing system that has been proportioned to carry all lateral forces plus sway effects, the behavior of beam columns is governed by local stability or strength and the design of these members can be based on the interaction equations using the sway prevented model. In the case of a structure that relies on the bending stiffness of beam to column joints to resist lateral forces, the column moment includes the sway effects (Equation 5.3), and thus, the behavior of individual members is influenced by the overall stability of the structure. Thus, the envelope provided by the sway prevented interaction equations no longer provides an adequate prediction of column end moment at ultimate load for all loading cases.

For h/r = 40 and G = 2 (Figure 5.2), the difference between the moment at ultimate load predicted by the analysis (upper solid line) and the envelope provided by the sway prevented interaction equations (upper broken and dashed lines) is quite small for all values of P/P_y. In this case, the stability effects are small and therefore, have less effect on the failure than the bending resistance of the total resistance of the column (as shown by the upper member). In Figures 5.2, 5.4, and 5.5, G represents the ratio of column to girder stiffness.

$$G = \frac{\Sigma I_c/h}{\Sigma I_g/L} \tag{5.8}$$

In the case of a more flexible column, h = 60r, as shown in Figure 5.4, the envelope no longer adequately predicts the ultimate strength for higher values of P/P_y. The same is true for Figure 5.5 in which the beam stiffness has been decreased, h = 40r, G = 10. In each

case, the presence of large second order effects causes a loss of lateral load capacity before the ultimate strength predicted by the interaction equations is reached.

The implication of this behavior is that structures designed using the PΔ procedure may fail due to overall stability before the individual members attain the ultimate moment predicted by the sway prevented interaction equations. This difference is only significant, however, when the sway effects become excessive. This condition may result from excessive deflections (the frame is too flexible) or excessive vertical loads applied to the storey or a combination of both.

To examine possible limits that could be imposed to avoid a stability failure at factored loads, an elastic analysis of the restrained column permitted to sway was performed. In this case, both beams and columns were assumed to be elastic. The solution was derived using the slope deflection equations. The resulting subassemblages were analyzed for various combinations of h/r, G, σ_y, and P/P_y.

Initially, a dummy lateral load was applied and a first order deflection and second order deflection were obtained for that load. For a given member size, end restraint, strength, and axial load, a lateral shear was calculated that would satisfy the interaction equations (Equations 3.19, 3.20, and 3.21). The second order deflection was then calculated at design loads to explore a relationship between the stability problem and the deflection index. The relationship between axial load, P/P_y, and the resulting deflection for the applied design load is shown in Figure 5.6 for various combinations of h/r, G, and σ_y.

As the axial load in the column increases, the limiting

bending moment specified by the interaction equations must decrease, thus, the lateral load that can be carried by the beam-column subassemblage must also decrease. As a result, the deflection obtained from the application of the design lateral load is found to decrease with increasing axial load (P/P_y) in Figure 5.6. Thus, there are values of axial load for which the difference between the ultimate column moment predicted by analysis and the ultimate column moment predicted by the sway prevented interaction equations is significant, yet the sway index is within the required limits. For example, when $h/r = 60$, $G = 2$, $\sigma_y = 36$, and $P/P_y = .7$, there is a significant difference between the column moment at failure and the limited predicted by Equation 3.20. The results of the elastic analysis shows a working load sway index $\Delta/h = 0.0016$. Thus, a stability problem can exist even when the deflection is well within suggested limits for the allowable deflection (24).

While it is likely that in a structure of practical and normal load distribution the stability problem may be overcome by designing the structure to a reasonable drift limit (51), there may be exceptional frames for which this criteria may not be sufficient. In the case of a structure subjected to large vertical loads and relatively small lateral loads, a stability problem can exist even if the sway index is satisfied. This situation would be particularly critical if design is governed by the vertical load only case, when relative lateral sways are small. One possible example of such a structure would be the case of a "leaned frame" in which a flexible frame is linked to a more rigid frame that is designed to carry all sway effects. In such a case, the deflection may be within specified limites, while the sway forces may be considerable.

To obtain a more generally applied limit, the nature of the stability problem must be examined. If a simple rigid frame subjected to vertical loads is perturbed by the application of a lateral load H, the structure will sway due to the lateral load an amount δ. The effect of the lateral load acting on the deflected shape is the equivalent of an additional shear, ΔH. This additional shear results in an additional sway $\Delta\delta$ which results in another additional effective shear. If the stability effects are small, an equilibrium position is obtained quite rapidly. If the stability effects are significant, the equilibrium position of the frame is more difficult to obtain without a significant number of iterations. To examine this phenomena, the approximate analysis outlined in section 5.2 was performed on a variety of elastic subassemblages. The number of iterations required to provide a difference between sway calculated from successive approximations of less than 3% was obtained. The results are shown in Figure 5.7 as number of interactions for convergence versus P/P_y for values of h/r, G, σ_y. When results are compared with Figures 5.2, 5.4, and 5.5, the onset of the stability problem corresponds to a rapid increase of the number of iterations required for convergence. In addition, the final deflection obtained by the approximate analysis was compared with the true second order deflection obtained when the sway term is included in the slope deflection equations. The apparent error in the approximate analysis increased rapidly as the number of iterations required for convergence increased as shown in Figure 5.7. Thus, there appears to be a correlation between the convergence of the second order analysis and the error introduced by using the sway prevented interaction equations to

predict the moment in a sway permitted column at ultimate load. For the examples shown, this error will be reasonably small if the number of iterations for a difference of successive approximations of 3% does not exceed 5.

If a stability problem exists, however, this process may be time consuming. In addition, if a direct second order solution is used, the existence of a stability problem may not be obvious. A more practical limit may be derived from the fact that the cause of the stability problem is the existence of sway effects (PΔ shears) that are large when compared to applied shears.

A ratio that is an indicator of the magnitude of the stability effects can be obtained from the results of a first order analysis. If the sum of shears applied to a storey is ΣH and the sum of additional sway forces applied to the storey (Equation 5.2) calculated from the first order analysis is $\Sigma H'$, then

$$R1 = \frac{\Sigma H'}{\Sigma H} \qquad (5.9)$$

This ratio is plotted in Figure 5.8 versus axial load P/P_y for various combinations of h/r, G, and σ_y of the elastic subassemblage. When compared with Figures 5.2, 5.4, and 5.5, and the rate of convergence in Figure 5.7, the onset of a stability problem can be predicted by a ratio, R1, of 0.50.

Thus, if the sway effects ratio, R1, is greater than 0.5 for any storey of a frame, a stability problem exists and convergence of the approximate PΔ analysis will be slow. An analysis of the convergence of the approximate PΔ method by MacGregor and Hage showed that

convergence to a difference of successive approximations of 5% will occur if $R1 \leq 0.55$ (51).

The ratio of sway effects, R1, may also be used as an indicator of the significance of second order effects to the analysis. When the values of this ratio for a structure are generally low, say 0.03, the error, if the second order effects are neglected completely, will be approximately 3% (51). Thus, when this ratio is less than a specified value for lower structures with small axial loads, there would be no need to perform a second order analysis or account for second order effects.

5.5 EFFECT OF AXIAL LOAD ON MEMBER STIFFNESS

Normally, the distribution of moments throughout a structure is determined on the basis of an elastic analysis which neglects the influence of column axial loads on the stiffness of the individual members. In the presence of an axial compressive force, the stiffness of a member is reduced (1). This reduction is a function of $\sqrt{PL^2/EI}$, where P denotes the axial load on the member, L the member length, E denotes the modulus of elasticity, and I the moment of inertia in the plane of bending.

The general slope deflection equation for an axially loaded member relating end moment at A to rotations at A and B, θ_A and θ_B, and a relative end movement, ρ, is given by (1).

$$M_A = \frac{EI_x}{L} (C\theta_A + S\theta_B - \rho(C + S))$$

where C and S are stability functions dependent on $\sqrt{PL^2/EI}$. If axial load P is assumed to be zero, then C = 4 and S = 2. To explore the

effect of axial load on the analysis of frames, the elastic subassemblage outlined in section 5.4 was analyzed using C = 4 and S = 2, and then, analyzed again with C and S depending on $\sqrt{PL^2/EI}$ for a variation of P/P_y, h/r, G, and σ_y. The results of these analyses are shown in Figure 5.9 as the percent error in deflection when C = 4 and S = 2 are used in the analysis versus the value $\sqrt{PL^2/EI}$.

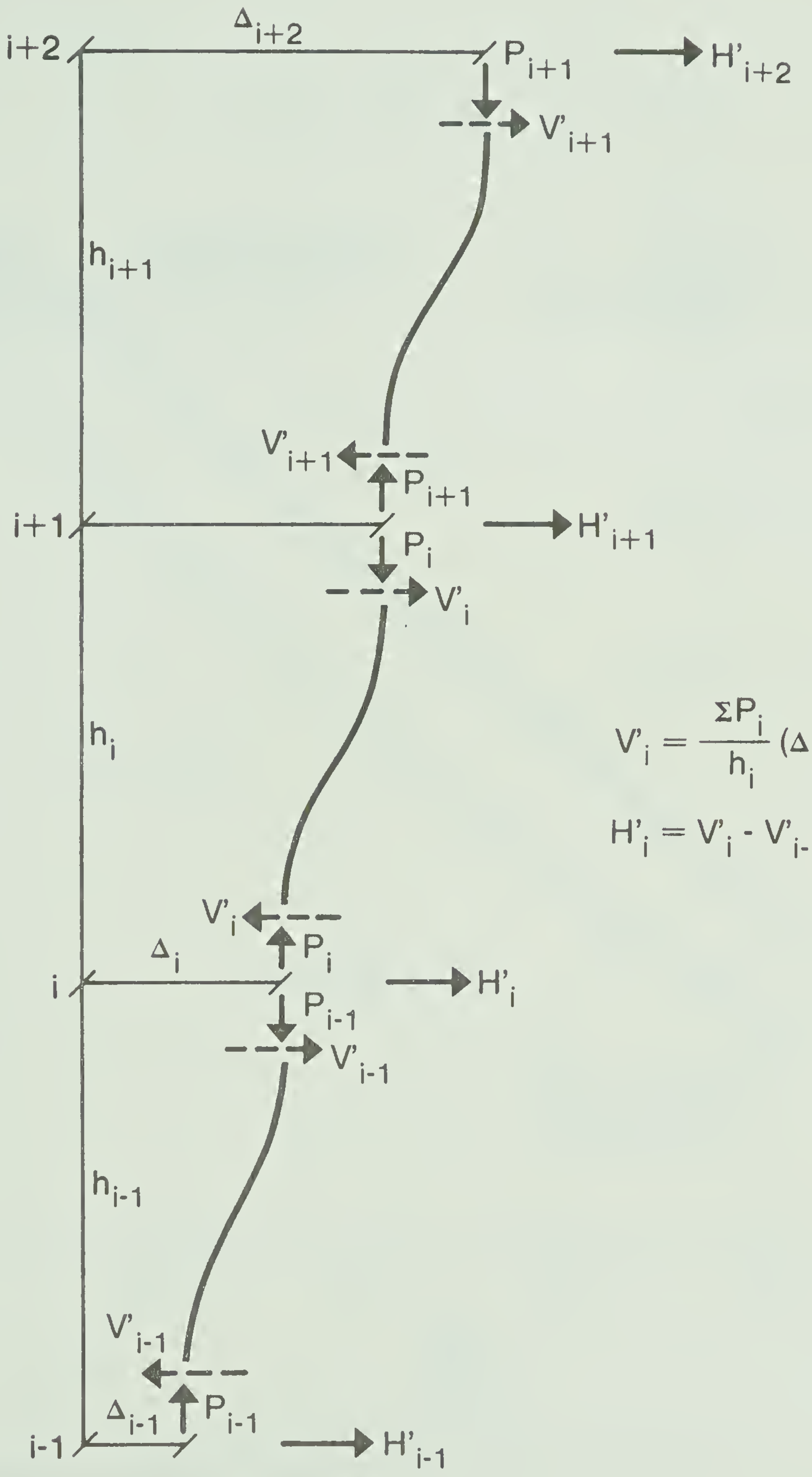
The errors were found to vary as the stiffness of the subassemblage varied. For the subassemblages studied, the error in calculated deflection was less than 2% if the value of $\sqrt{PL^2/EI}$ was less than 1.2. For values of $\sqrt{PL^2/EI}$ greater than 1.2, consideration must be given to including the effects of axial load on member stiffness.

For most structures of practical proportions, the value of $\sqrt{PL^2/EI}$ is considerably less than 1.2. For the design example discussed in chapter 7, the use of C = 4 and S = 2 was found to produce an error of considerably less than 1% in both sways and member forces and moments when compared with an exact analysis.

5.6 SUMMARY

The design of beam-columns using the PΔ procedure has been outlined. An approximate method for obtaining a second order analysis from the results of a first order analysis has also been presented. A comparison between limits predicted by the PΔ Procedure and the results of a second order elastic plastic analysis of beam columns shows a good correlation, except when failure is governed by a stability failure. This stability problem can be avoided by a simple check of the magnitude of the sway effects during the analysis of the structure.

Since the accuracy of the PΔ Procedure depends on the accuracy of the prediction of the deflected shape of the structure, all effects that can increase the deflections, hence, the second order shears must be considered. The effect of axial load on column stiffness and inelastic behavior of members have been considered. The effect of axial load on the resulting deflection was found to be less than 2% if $\sqrt{PL^2/EI}$ is less than 1.2. The effect of inelastic action will require further analysis and will be examined in chapter 6. It is suggested, however, that all effects that may be significant to the calculation of deflection must be considered when a PΔ Procedure is employed. Such effects may include effect of column shortening, effect of shear on joints, and any effect that may cause significant deflection.



$$V'_i = \frac{\Sigma P_i}{h_i}(\Delta_{i+1} - \Delta_i)$$

$$H'_i = V'_i - V'_{i-1}$$

Figure 5.1 Sway Forces Due To Vertical Loads

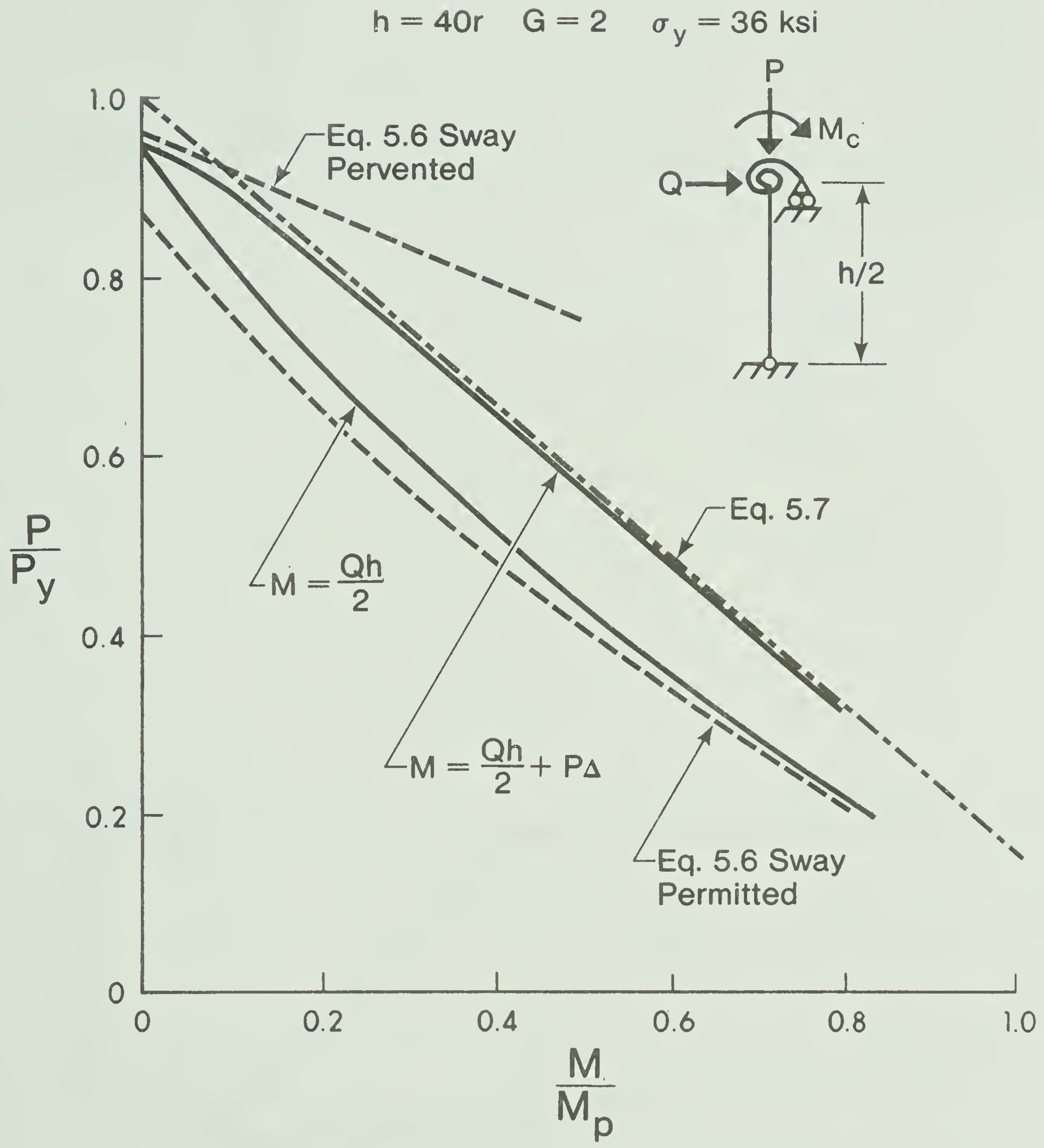


Figure 5.2 Interaction Relationships, h/r = 40 G = 2

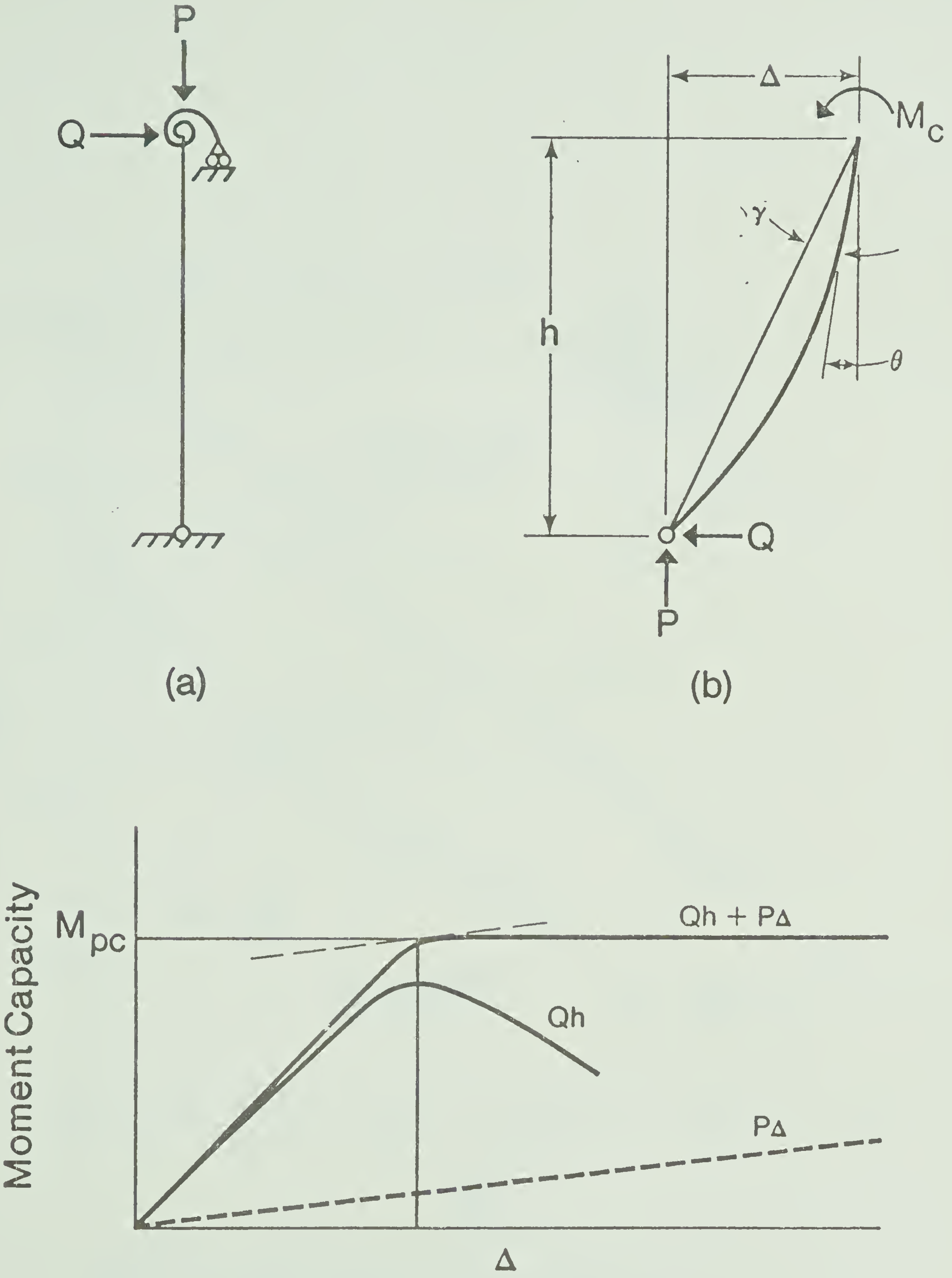


Figure 5.3 Loss of Load Carrying Capacity Due to PΔ's

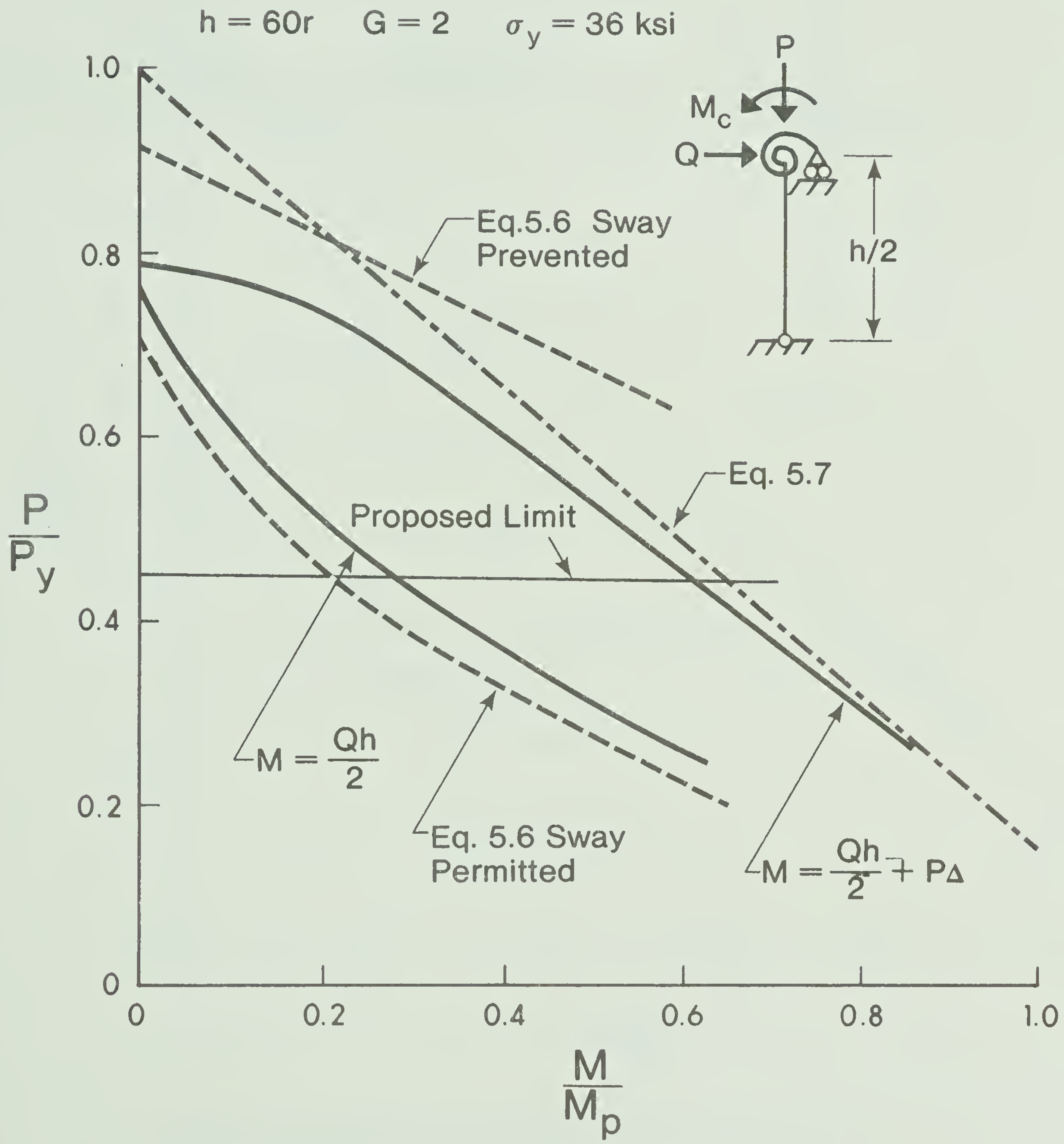


Figure 5.4 Interaction Relationships, h/r = 60, G = 2

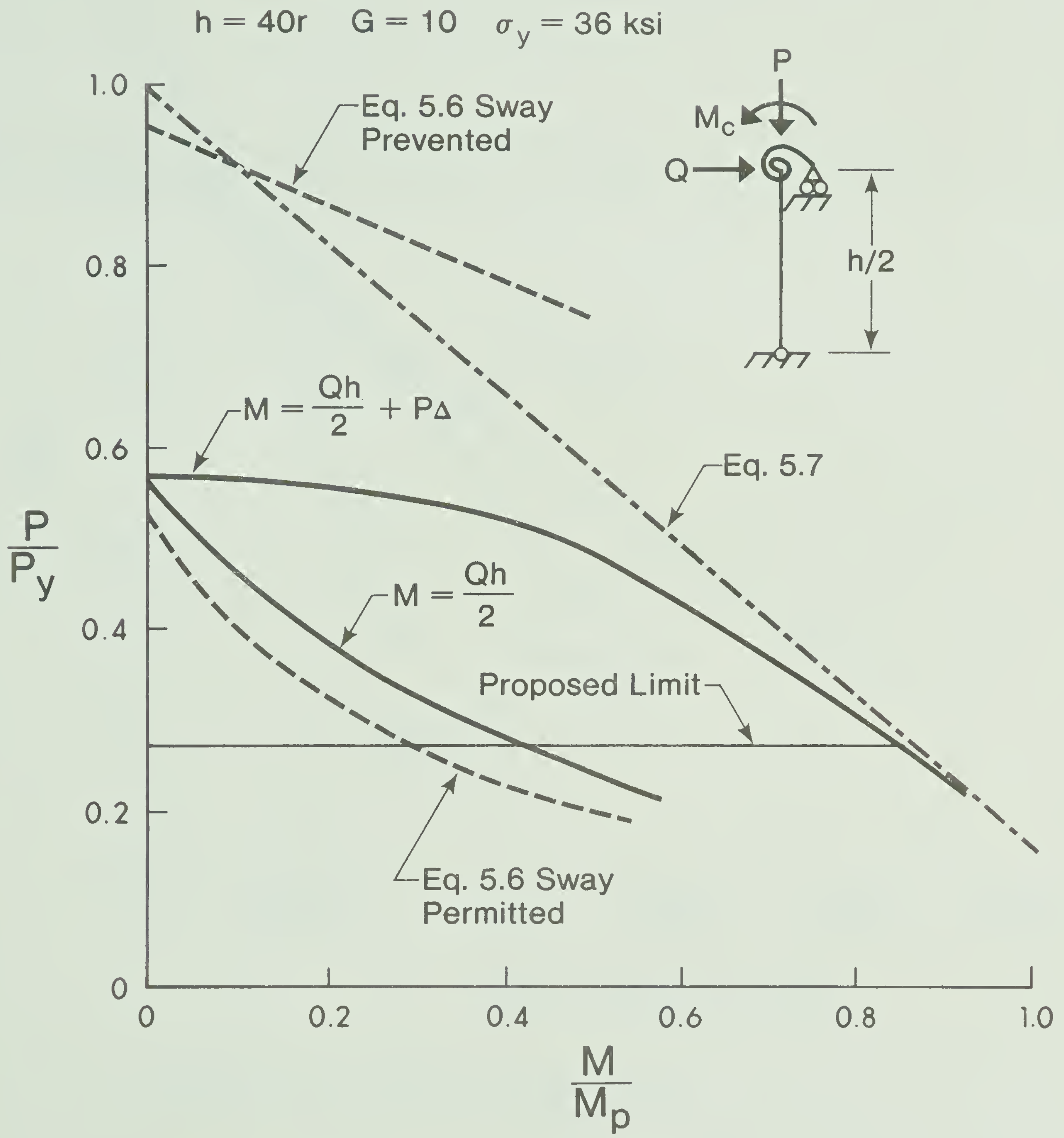


Figure 5.5 Interaction Relationships H/R = 40, G = 10

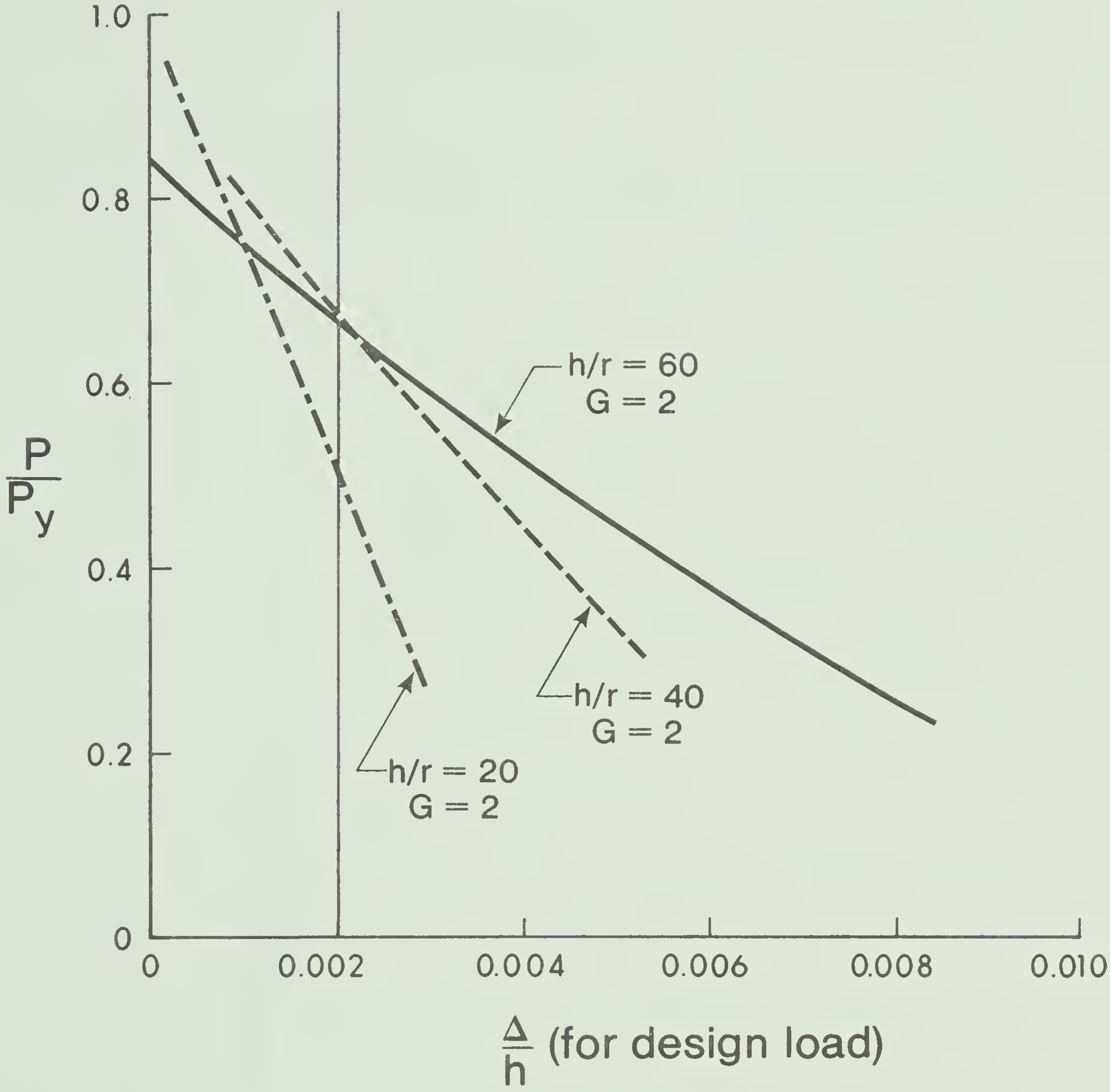


Figure 5.6 P/Py vs Sway At Specified Load

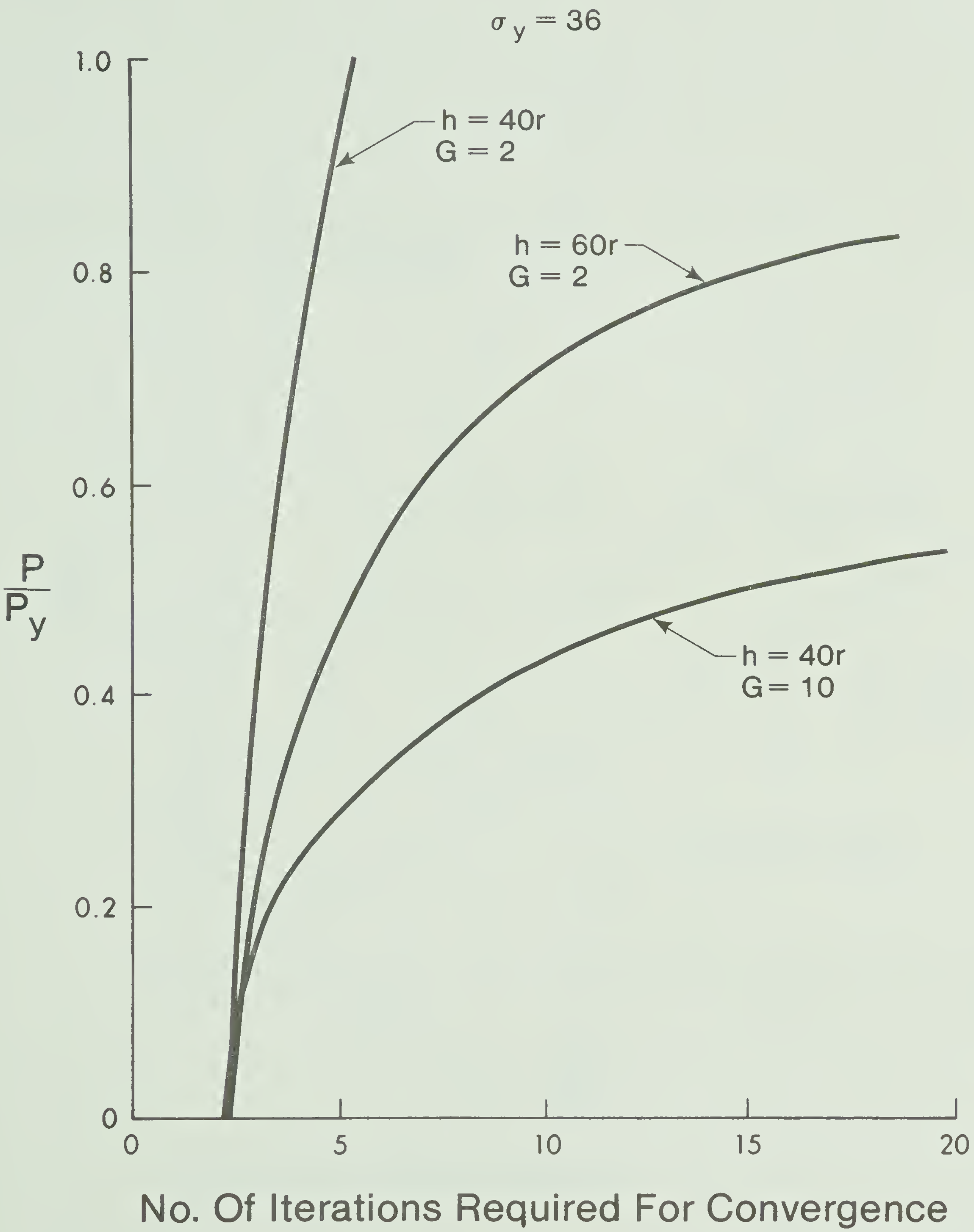


Figure 5.7 P/Py vs No. Of Iterations For Convergence of P-Δ Analysis

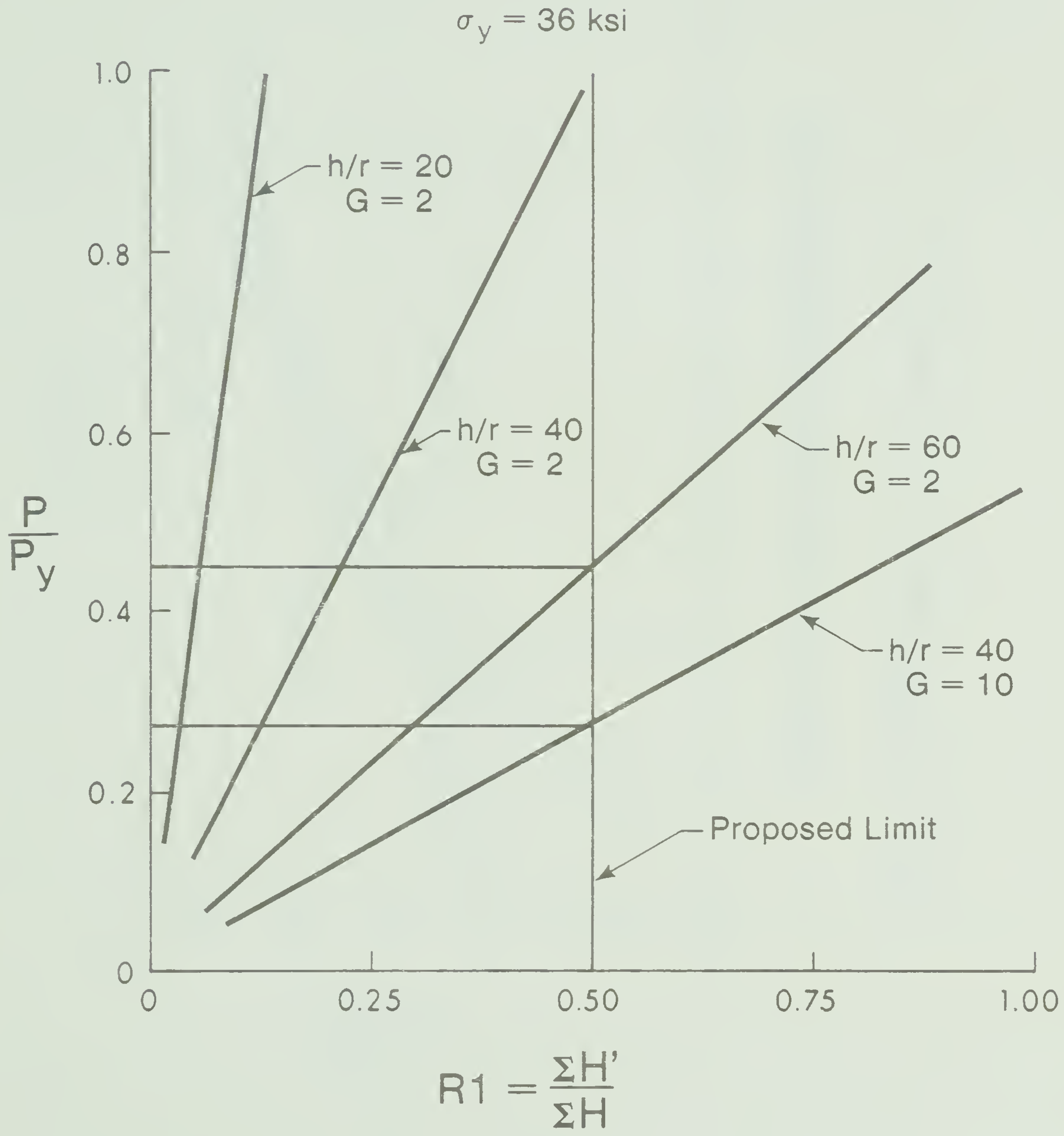


Figure 5.8 P/Py vs Sway Effects Ratio

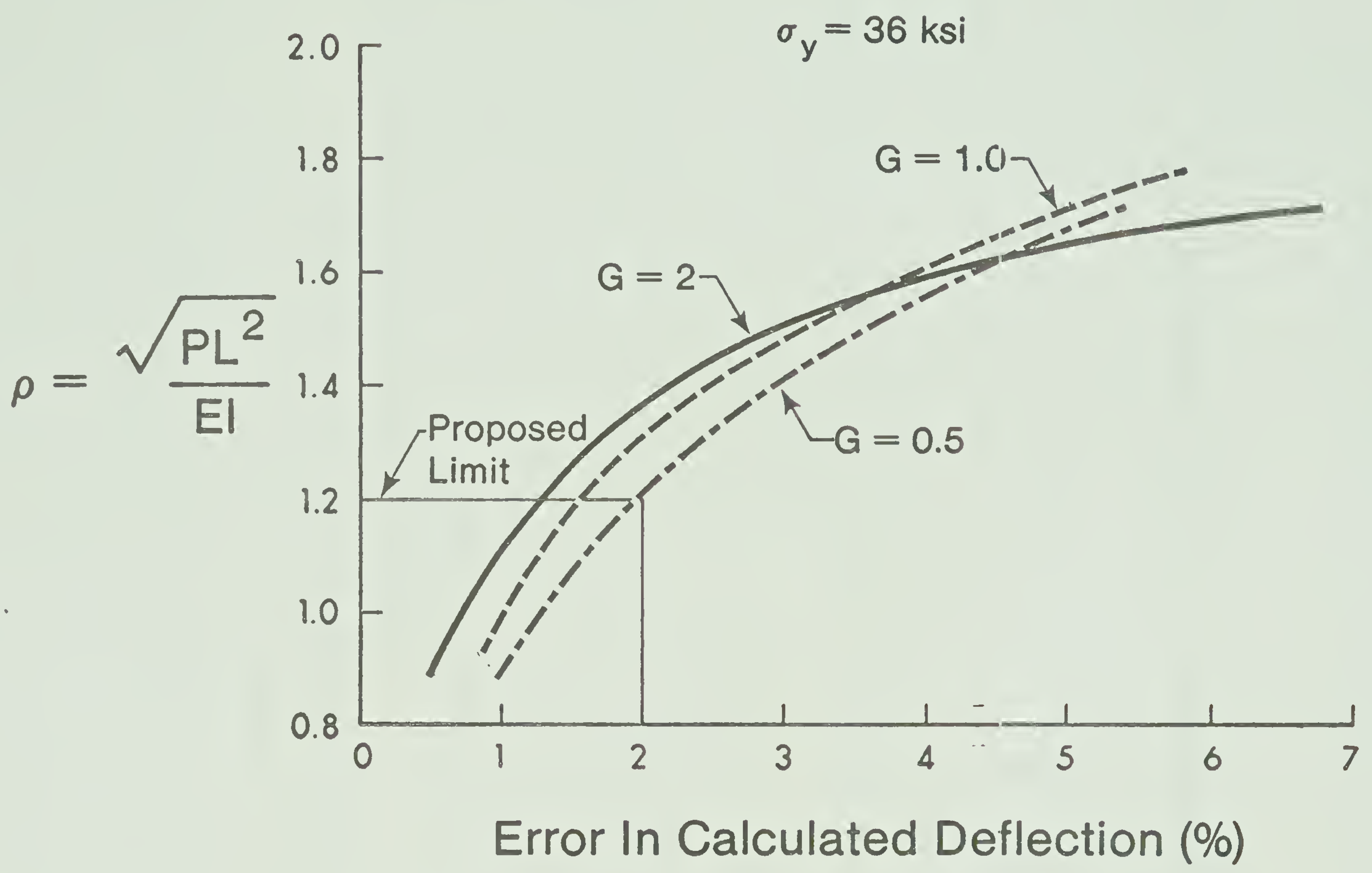


Figure 5.9 $\sqrt{PL^2/EI}$ vs Error In Calculated Deflection

CHAPTER VI

EFFECT OF INELASTIC BEAM RESPONSE ON BEAM COLUMN BEHAVIOR

6.1 INTRODUCTION

When the PΔ design procedure is used to design columns in a structure as outlined in Chapter V, it is essential that the model used in the analysis predicts the behavior of the structure accurately. In the traditional design procedure, the results of the analysis have been adjusted to account for the differences between actual frame response and the behavior predicted by the analysis. When the PΔ procedure is used, this adjustment is not applied.

The analysis used to predict the behavior of the structure normally assumes a perfectly elastic plastic response for the restraining effect of the beams on the columns as shown by the solid line in Figure 6.1. The effect of residual stresses and gradual penetration of yield results in a rounding off of the moment-curvature curve for a beam as shown by the dashed line in Figure 6.1(43).

The resulting restraint provided by the beam to the column is less than predicted in an analysis that assumes the beam is elastic until the plastic moment of the beam or columns is reached. Consequently, the results of the analysis may overestimate the stiffness of the structure and result in an unconservative prediction of the second order effects at the limit state. The effect of gradual plastification of the beam was discussed by Daniels and Lu(35). While they suggested that the shear resistance of the storey would be reduced, they also

suggested that this reduction would be insignificant when the assumptions on which the subassemblage method are based or considered (35). The results of large scale tests has tended to confirm this assertion at present (16,17,18,20,21). Since an analysis that will predict the effect of gradual plastification of restraining beams on the behavior of beam columns has not been developed, the significance of this effect cannot be confirmed.

The analysis used in the previous chapters has assumed an elastic restraining function at the upper joint in the column. In this Chapter, a second order analysis that predicts the behavior of beam columns restrained by both elastic-plastic and inelastic restraining functions will be developed. The behavior predicted by this analysis will be compared to other analyses and experiments. The differences between the behavior of columns with elastic-plastic and inelastic restraining functions will be considered with emphasis on the prediction of the limit state of the column.

6.2 INELASTIC ANALYSIS OF SUBASSEMBLAGES

The model used to predict the effects of inelastic beam restraints on beam column behavior is based on the beam column subassemblage as outlined in Chapter 3 and shown in Figure 6.2. To include the effects of gradual plastification, a numerical integration procedure, outlined in Reference 13, is used to model inelastic beam behavior. Boundary conditions are satisfied by compatability conditions and equilibrium equations applied. The material properties for the beams are provided by the moment-curvature relationship that is specified for each particular problem.

The far ends of the beams, B and C, are assumed to be restrained by elastic springs. The stiffness of each spring is specified by the relative stiffness of the joint, B or C, with respect to joint A. Thus, a rotation at joint A, θ_A, is accompanied by a rotation at joint B, θ_B, where

$$\theta_B = f(\theta_A) \tag{6.1}$$

The function of Equation 6.1 may be determined using a procedure outlined by Daniels and Lu(35) or the joints may simply be assumed to have equal stiffness. In the case where the far end is pinned, the solution is simplified by setting the moment at the far end of the beam to zero and allowing the far end of the beam to rotate freely.

The forces acting on the beam and the column of the subassemblage are shown in Figure 6.3. To obtain the relationship between applied lateral shear and joint deflection, the joint A is given an arbitrary rotation θ_A. The equilibrium of the beam is enforced by a summation of moments about A in Figure 6.3a to calculate V_{AB}:

$$V_{AB} = \frac{(M_{AB} + M_{BA} + (Q_{AB} \quad L_{AB}{}^2)/2)}{L_{AB}} \tag{6.2}$$

To perform the integration, the beam is broken up into a number of segments, N. The size of each segment should be approximately equal to the radius of gyration of the beam, (13). The bending moment at the centre of each segment is calculated by

$$BM = V_{AB}X - (Q_{AB} \; X^2)/2 - M_{AB} \tag{6.3}$$

where X is the distance from the end of the beam to the centre of the

segment. Knowing the bending moment in each segment, the curvature of each segment can be found from the specified moment-curvature relationship.

The moment-curvature relationship used at this point will define the type of restraint provided by the beam to the column. In the computer program developed for this analysis, the moment-curvature relationship is described by a series of points joined by straight line segments. A horizontal segment of the M-ϕ curve is described by two points with a finite, but very small slope between them. Essentially, any moment-curvature relationship can be specified at this point. Examples of elastic, elastic-plastic, and inelastic M-ϕ curves are shown in Figure 6.4 and the corresponding input data for each curve are listed in Tables 6.1, 6.2, and 6.3.

The curvature obtained from this M-ϕ curve is used to calculate the deflection at the end of the segment in the equation (13):

$$V_i = V_{i-1} + \alpha_{i-1}\,\delta_i - \frac{\delta_i^2\phi_i}{2} \tag{6.4}$$

where:

V_i = deflection at the end of the i^{th} segment

V_{i-1} = deflection at the end of the $i - 1^{th}$ segment

α_{i-1} = the slope at the end of the $i - 1^{th}$ segment

δ_i = the length of the i^{th} segment

ϕ_i = the curvature of the i^{th} segment.

The slope at the end of any segment, α_i, can be found by, (13);

$$\alpha_i = \alpha i - 1 - \delta_i \phi_i \tag{6.5}$$

Using Equations (6.4) and (6.5), the beam is integrated to obtain the slope and deflection at the far end. As the beam is supported at the far end, the deflection must be zero. This deflection can be altered by varying the near end joint moment, M_{AB}. The slope at the far end of the beam is controlled by the rotation of joint B. Thus, having specified a relationship between the rotations of joints A and B, for a given rotation at A the required joint rotation at B, θ_B, can be calculated. The slope at the far end of the beam must then be equal to the rotation of the far end joint, θ_B. If this condition is not satisfied, the far end moment M_{BA} is adjusted until the condition is satisfied.

A satisfactory solution for the beam is obtained by adjusting the end moments M_{AB} and M_{BA} until both of the end conditions are satisfied. If these conditions are satisfied, then the beam is compatible with the specified boundary conditions and is in equilibrium.

Each beam is considered separately and a set of joint moments M_{AB} and M_{AC} are obtained. Then, for equilibrium of the joint,

$$- M_{AD} + M_{ext} + M_{AB} + M_{AC} + (V_{AB} - V_{AC})d/2 = 0 \tag{6.6}$$

if $M_{ext} = -M_{AD}$, that is, the columns share the resisting moment, then:

$$M_{AD} = - (M_{AB} + M_{AC} + (V_{AB} - V_{AC})\ d/2)2 \tag{6.7}$$

where d is the column width.

Having calculated the end moment for the column, M_{AD}, using Equation (6.7), the corresponding end rotation for the column, γ, can be obtained using a moment-rotation curve. Examples of moment-rotation

curves are given in Reference 25. These curves were developed using the numerical integration procedure outlined in Reference 13.

The sway that results from the joint rotation θ_A and the column end rotation γ is found by

$$\Delta/h = \theta_A - \gamma \tag{6.8}$$

In the case when the column has an initial sway due to manufacturing or erection, this sway may be included in Equation 6.8 as (Δ/h) imp

$$\Delta/h = \theta A - \gamma + (\Delta/h)\text{imp} \tag{6.9}$$

The shear resistance of the subassemblage, Q, is obtained by a summation of moments about A in Figure 6.3b.

$$Q = M_{AD} - P\,\frac{\Delta}{h} \tag{6.10}$$

By repeating this procedure for different joint rotations θ_A, the full load-deflection curve may be obtained for a given subassemblage.

6.3 COMPUTER PROGRAM

The analysis described in the preceding section was programmed in Fortran IV for the IBM 360/67 system. In this section, the logic of the program and the function of each subroutine is briefly described. The program is outlined in Appendix B; the nomenclature of the program in section B.1, the flow diagrams in section B.2, the input data in section B.3 and the program listing is presented in section B.4.

The analysis proceeds in the following manner:

1. The beam properties are input, including section properties, length, loading and a moment-curvature relationship as graph co-ordinates.

2. The column properties are input, including section properties, length, yield stress, P/Py and a column moment vs end rotation relationship as graph co-ordinates.
3. The initial joint rotation, joint rotation increments, initial imperfection, relative joint rotations are input.
4. All initial calculations required in the program are performed, such as, beam and column stiffness, and plastic moments, initial fixed end moments and other values.
5. Using the fixed end moments as an initial guess, the left hand beam is integrated, boundary conditions are checked, end moments are adjusted as required, and the process is repeated until boundary conditions are satisfied.
6. Step 5 is repeated for the right hand beam. The signs used for the integration of the right hand beam are altered as the integration direction has been reversed.
7. After both beams are satisfied, the column end moment is calculated. The column end rotation is calculated from the moment end rotation relationship.
8. Column deflection is calculated, and column shear is calculated. Column deflection and shear are output along with beam end moments and column moment.
9. Joint rotation is increased and steps 5 to 8 are repeated. The end moments at the conclusion of step 8 are used as initial trial values.
10. The program continues until the column reaches plastic moment or a specified number of joint increments have been repeated.

The main program controls the analysis and calculates and

outputs final results. Subroutine READ inputs and echo checks data. Subroutine CALC calculates beam and column properties required by the analysis. Subroutine BEAM checks boundary conditions and alters end moments to satisfy boundary conditions. Subroutine SUM integrates the beam using the moment curvature curve provided to find far end rotation and deflection. Subroutine COLUMN uses the moment-rotation data to calculate column end rotation.

6.4 COMPARISON WITH EXISTING ANALYSIS

To confirm the logic and accuracy of the proposed analysis, a comparison was made between the results given by Daniels and Lu (35) and the results of the proposed inelastic analysis. For comparison, a subassemblage was used with h/r = 24 and beams that provide an equivalent resisting moment of $M_r = 250\ \theta M_{pc}$. To simplify the analysis, pin connections were assumed at the far ends of the beams. The results of these analyses are shown in Figure 6.5 with the Daniels and Lu analysis shown as the solid curve and the proposed analysis as points on the curve. There is an excellent agreement between the results of these two analyses. This difference results from the calculation of the beam resisting moment at the column centreline. Daniels and Lu used the centre to centre column distance to approximate the resisting moment, (35), while the proposed analysis calculates the true column centreline moment by including the beam shears in Equation (6.6). In a sway frame, the column shears effectively increase resisting moment, while the effect of Daniels' and Lu's assumption is to decrease resisting moment.

If a beam load is included, and elastic-perfectly plastic

beam response is specified, the curves in Figure 6.6 are obtained. The lower solid curve is obtained using Daniels' and Lu's analysis (35), while the upper dashed curve represents the results of the inelastic analysis procedure. The location of the plastic hinge in the beam differs in each case. Daniels and Lu assume the plastic hinge occurs at the column face (35), while the use of numerical integration in the inelastic analysis procedure results in the formation of a hinge a finite distance from the column face. Tests have shown the latter assumption may be more realistic (37).

A comparison between the results of the inelastic analysis procedure and the results of subassemblage analyses (35) shows a good agreement when elastic and elastic-plastic beam responses were specified. Differences between the analyses result from the apparent locations of plastic hinges and hence, the point on the load-deflection curve at which the beam stiffness is reduced due to hinge formation. The hinge location in the proposed analysis does not appear to be inconsistent with actual frame behavior (27).

6.5 COMPARISON WITH EXPERIMENTAL DATA

The inelastic analysis was used to predict the behavior of a number of experiments performed by Kim and Daniels (21,22). The first set of experiments included tests on restrained columns permitted to sway (21). The second set of experiments included two unbraced subassemblages (22).

For these analyses, the inelastic beam response shown by the moment-curvature relationship in Figure 6.1 was specified (1).

These M-ϕ curves were applied with and without the effects of strain hardening. While the typical moment-curvature relationship for a wide-flange beam should include the effects of strain hardening, the reduction of moment carrying capacity brought on by local or lateral buckling must also be considered.

For the sake of the analysis, the point loading used in the experiment was approximated by an equivalent uniformly distributed load. The intensity of this applied load was chosen to provide end moments consistent with the end moments resulting from the point loads in the experiment.

The results of test RC-1, a windward restrained column are shown in Figure 6.7, (21), as the solid line joining points. The broken line is an analysis using an elastic-plastic beam response. The dashed line is the result of an inelastic beam response, (21). The analysis is only adequate as a prediction in this case. The results of this experiment are questioned, as during the experiment, one of the rollers was found to be jamming. In addition, the frame was found to have an initial out of plumb that was not measured. The analysis was corrected for this out of plumb by approximating the initial Δ/h. The behavior predicted by the inelastic analysis is more consistent with the actual behavior than the behavior predicted using an elastic perfectly plastic beam response.

The results of test RC-2, an interior restrained column are shown as the solid line in Figure 6.8 (21). The results of an inelastic analysis are shown as the dashed line in the Figure. The analysis provides an excellent prediction of behavior. Discrepancies between the

predicted and actual behavior beyond the ultimate load appear to be due to the difficulty in predicting the effect of strain hardening and local buckling in both beam and column near ultimate load.

The results of test RC-3, a leeward restrained column are shown as the solid line in Figure 6.9 (21). The prediction based on the inelastic analysis is shown as the dashed line in the Figure. The results of the inelastic analysis provide a reasonable prediction of the initial portion of the load deflection curve. There are, however, apparent discrepancies in the later stages of the curve. Again, these discrepancies are likely due to the effects of strain hardening. In the report on the test results, the restrained column was found to attain a moment of M/Mpc = 1.25.

The results of an experiment of an unbraced subassemblage are shown in Figure 6.10(22) as the solid curve. This subassemblage, SA-1, was designed to simulate the top stories of a building. An analysis was performed by breaking the subassemblage into three restrained columns and specifying an inelastic beam response. The column load in each case was assumed to be the column load at failure. The results of these analyses were summed to give the resulting broken curve in Figure 6.10. This analysis provided an excellent prediction of actual behavior. The slight discrepancy in the ultimate load (1%) is likely due to the difficulty in predicting the effects of strain hardening.

The results of experiment SA-2 are shown as the solid curve in Figure 6.11(22). In this case, the subassemblage was designed to approximate the behavior of the lower stories of a tall building. The prediction based on the proposed inelastic analysis shown as a solid

curve in Figure 6.11 provides a close agreement with the experimental results. Analyses performed by Kim and Daniels are shown by the dashed and broken lines in the figure (22).

6.6 BEHAVIOR OF INELASTIC BEAM COLUMN SUBASSEMBLAGES

6.6.1 EFFECT OF LOAD SEQUENCE

The behavior of the subassemblage outlined in Section 6.2 will depend on the nature and sequence of loads applied to the members of the subassemblage. The subassemblage may be subjected to beam loads, w, column axial loads, P, and a lateral sheer, H, as shown in Figure 6.2. When using Limit States Design Procedures, the critical load for the subassemblage is defined as the load or combination of loads at which the factored resistance of any member is equal to the effect of factored loads (24). For beams, this limit is usually defined by yield or plastification of the beam. For columns, the failure may be due to local yield or plastification or an overall stability failure.

Theoretically, if a perfect symmetrical subassemblage is subjected to gravity loads only, the limit state may be reached due to plastification of the beam or column or a buckling of the column. Concern has been raised of the effect of gradual penetration of yielded regions in both beams and columns on the buckling of restrained columns (7,33). This problem was discussed in chapter 3 and is a source of continued research (44,45).

Real structures, however, are not perfect. Initial imperfections and load eccentricities produce sway (P-Delta) forces. The actual behavior of the subassemblage is not unlike that of a subassemblage

subjected to a lateral load. For a sidesway subassemblage, the limit state may be reached due to local plastification of the beam or column or a loss of load carrying capacity due to overall instability of the subassemblage. The point of overall instability is reached when the lateral shear resistance of the restrained column is less than the sway effects plus any applied lateral shear.

The problem then is to define under what load combination or sequence a gradual plastification can significantly affect the occurrence of the limit state. Initially, only non-proportional loading will be considered. A constant vertical gravity load shall be applied and the horizontal shear will be varied. This type of load sequence is more likely to occur naturally as large variations of gravity load (snow or occupancy) do not normally occur simultaneously with large variations of lateral load (wind or earthquake).

To examine the effect of loading on the behavior of inelastic subassemblages, a subassemblage with h/r = 30 and G = 1.5 was analyzed using an elastic perfectly plastic beam response and then reanalyzed with an inelastic beam response as defined by the beam moment curvature relationship (see Figure 6.1). For these analyses, the beam loads were varied up to a maximum of w = 0.6 wp, where wp is the load that results in a beam failure under gravity loads only. The beam loads were held constant while the sidesway response of the subassemblage was obtained using the analysis outlined in Section 6.2.

The results of an analysis of the subassemblage with w = .6 wp are shown in Figure 6.12. The elastic plastic analysis is shown as the solid line, the inelastic analysis is the dashed line.

The results of each analysis are essentially identical except for the range of loads at which the beams form leeward hinges. The effect of these hinges are apparent first in the inelastic analysis as the sub-assemblage response softens due to gradual penetration of the yielded zone. After the hinges have formed in the elastic-plastic model, the two curves converge again. The ultimate loads obtained are nearly identical for each analysis. The ultimate load for the inelastic analysis was actually slightly greater when strain hardening was included in the beam response. A similar analysis for w = 0.45 wp is shown in Figure 6.13. Again, the responses predicted by both elastic-plastic and inelastic analyses are similar except for range of loads at which inelastic action occurs. There is virtually no difference between ultimate loads.

For each of these analyses, the ultimate load resulted from an overall stability failure rather than a local strength failure. The plastification of the beam affected the failure by reducing the restraining function at the joint. A comparison of limit states for each analysis is extremely difficult as the limit state of the inelastic restrained subassemblage is quite difficult to define. If, however, the limit state is defined as full plastification of the beam, the limit state as predicted by the inelastic model would be greater than that assumed for an elastic-plastic model.

The specific concern, however, is to determine the effect of inelastic beam response on the behavior of a beam column at the limit state. While there was a local divergence of the curves predicted by the different beam responses, the ultimate loads predicted by each ana-

lysis were very close. This lack of distinction between the results predicted by differing beam response relates to the ultimate conditions. In each case, beam hinges were fully developed and the restraining moments provided by the beams were essentially equal to the beam plastic moment. Thus, when considering the ultimate load, it matters not how the beam developed a plastic moment, whether by gradual penetration of yield or by developing a plastic hinge "instantaneously", after plastification has taken place the restraint provided by the beam in each case is the same. Since in each case, the restraining effect of the beam on the column is effectively the same, there will not be a significant difference in overall column behavior.

As a result, when considering the restraining effect of an inelastic beam on a column, it is important to apply the load condition that will significantly affect the behavior of an inelastically restrained column. If the restraining beams are fully elastic or fully plastic when the column attains ultimate load, the fact that the beam behaves inelastically during its loading history will have little effect on the column response. If, however, the beam is in an inelastic state, that is partial penetration of the yielded zone at or near the ultimate load for the subassemblage, the resulting increase in the deflection may lower the apparent ultimate load due to an increase in second order effects. Thus, when the effect of inelastic beams are studied, the beam loads should be adjusted to reflect inelastic beam response at the ultimate load of the subassemblage.

This critical condition may be attained using both proportional and non-proportional loading. Since the range of loads at which

this critical condition can develop is small, there will be essentially no distinction between the results obtained using each type of loading. For ease of obtaining this loading condition, non-proportional loading will be used.

6.6.2 EFFECT OF FRAME STIFFNESS

As outlined in the previous section, the effect of inelastic beam response is likely to be more pronounced if the beam is inelastic, but not plastic at the point when the ultimate load of the subassemblage is reached. In addition, this effect is likely to be more pronounced when the sway effects are greater. To study this behavior, subassemblages of various values for h/r, G, P/Py with σy = 44 ksi were analyzed using both elastic-plastic and inelastic beam responses. Typical results of these analyses are presented.

The subassemblages used for these analyses were symmetric beam-column subassemblages with the far ends of the beam pinned. While the use of beams with pinned ends reduced computing time considerably, the nature of the response of the subassemblage was not altered. In each case, the development of a plastic hinge in the beam gradually reduces the restraint provided to the column joint. The beam loads were selected by successive trials to produce an inelastic beam response without full plastification at the critical load for the subassemblage. For this study, significant strain hardening was not included.

The results of the analyses for h/r = 20, G = 2 are shown in Figure 6.14 for P/Py = 0.7. The curves for each type of analysis were almost identical. Similar results were obtained for other values of

P/Py.

The results of the analyses for h/r = 40, G = 2 are shown in Figure 6.15 for P/Py = 0.7. Again, the behavior predicted by both elastic-plastic and inelastic analyses were almost identical. There was a slight difference in ultimate load obtained from the analysis, but this difference was relatively insignificant.

The results of the analyses for h/r = 60, G = 2 are shown in Figure 6.16 for P/Py = 0.6. In this case, there was a significant difference between the ultimate load predicted by inelastic beam behavior and the elastic-plastic beam behavior. The discrepancy between ultimate strengths predicted by inelastic and elastic-plastic analyses became even more pronounced for h/r = 40, G = 10 and P/Py = 0.5 as shown in Figure 6.17.

Thus, the difference between a column's response to lateral load when restrained by an elastic-plastic beam or an inelastic beam varies as the column slenderness, the restraining function, and the axial load. The softening effect of gradual penetration of yielding was found to be significant only beyond the stability limit established in Chapter V. If the sway effects in a frame are significant, the inelastic response of beams can significantly reduce the ultimate strength of the frame (and hence, the limit state of the column). If the sway effects are not significant, the effect of beam yielding is not significant.

6.6.3 BEHAVIOR OF A ONE-STOREY BEAM COLUMN SUBASSEMBLAGE

In the previous sections, the response of simple beam-column

subassemblages were analyzed to examine the effects of inelastic beam restraint on the stability of beam-columns. In real structures, the overall stability of a planar bent is a function of all the separate responses of beam-columns summed together. In this section, the effect of local inelastic behavior on the overall performance of the frame bent will be discussed. This behavior is particularly important when using plastic analysis for the design of a structure.

To study the overall response of a frame, the subassemblage used in the analysis by Daniel and Lu (35) was considered (see Figure 6.18). The storey was broken up into four basic subassemblages. Each subassemblage was analyzed using the program outlined in Section 6.3 with the beam moment-curvature relationship defined by (a) elastic-plastic behavior; (b) inelastic behavior; (c) inelastic behavior with strain hardening as shown in Figure 6.1. The results of the separate subassemblages were then summed to obtain an overall load deflection response for that storey of the frame.

When the elastic-plastic response is compared to the inelastic response, as shown in Figure 6.19, there is a slight difference only near ultimate load. If a strain hardening function is included in the beam response, as shown in Figure 6.20, there is no apparent difference between ultimate loads. In each case, the load for formation of the first hinge, which is effectively the limit state for this portion of the structure, was found to be effectively the same for each type of analysis.

Thus, when overall frame behavior is considered, local losses in frame stiffness due to inelastic action are often compensated for by

the behavior of other members in the frame and the effects of strain hardening. A general conclusion for all structures, particularly those designed using plastic analysis, would require considerably more study of this point, possibly including large-scale tests.

6.7 SUMMARY

In this chapter, a procedure was developed to analyze inelastic beam-column subassemblages restrained by inelastic beams. An inelastic beam is defined as a beam subjected to gradual penetration of the yielded zone, including the effects of residual strains. The results predicted by this analysis were confirmed by a comparison with the results of various tests. Unfortunately, only one of the tests available contained significant inelastic behavior. The proposed analysis did provide significantly better predictions of the behavior of restrained columns permitted to sway.

The analysis was then used to predict the effect of inelastic beam response on the behavior of various beam-column subassemblages. The inelastic beam behavior was found to 'soften' the response of these subassemblages for a short period of load history, but only effect the ultimate strength of 'flexible' columns when the failure of the column accompanied the inelastic behavior.

When the behavior of a frame that included several restrained columns was considered, there was little difference found between the response of an elastic-plastic analysis and an inelastic analysis that included strain hardening.

TABLE 6.1 - ELASTIC PLASTIC
MOMENT - CURVATURE DATA

M/MP	ϕ/ϕ P
0.0	0.0
0.9990	0.9990
1.0000	11.0000
1.1000	800.0000

TABLE 6.2 - INELASTIC
MOMENT - CURVATURE DATA

M/MP	ϕ/ϕ P
0.0	0.0
0.6260	0.6260
0.7910	0.8040
0.8550	0.8940
0.8840	0.9480
0.8990	0.9830
0.9430	1.1620
0.9750	1.5200
0.9840	1.8800
0.9940	3.1300
0.9985	4.9100
0.9995	7.6000
1.0000	10.3000
1.0010	28.1000
1.1800	800.0000

TABLE 6.3 - INELASTIC - STRAIN HARDENING
MOMENT CURVATURE DATA

M/MP	ϕ/ϕ P
0.0	0.0
0.6260	0.6260
0.7910	0.8040
0.8550	0.8940
0.8840	0.9480
0.8990	0.9830
0.9430	1.1620
0.9750	1.5200
0.9840	1.8800
0.9940	3.1300
0.9980	4.9100
0.9990	7.6000
1.0000	10.3000
1.4440	28.1000
1.6700	178.0000

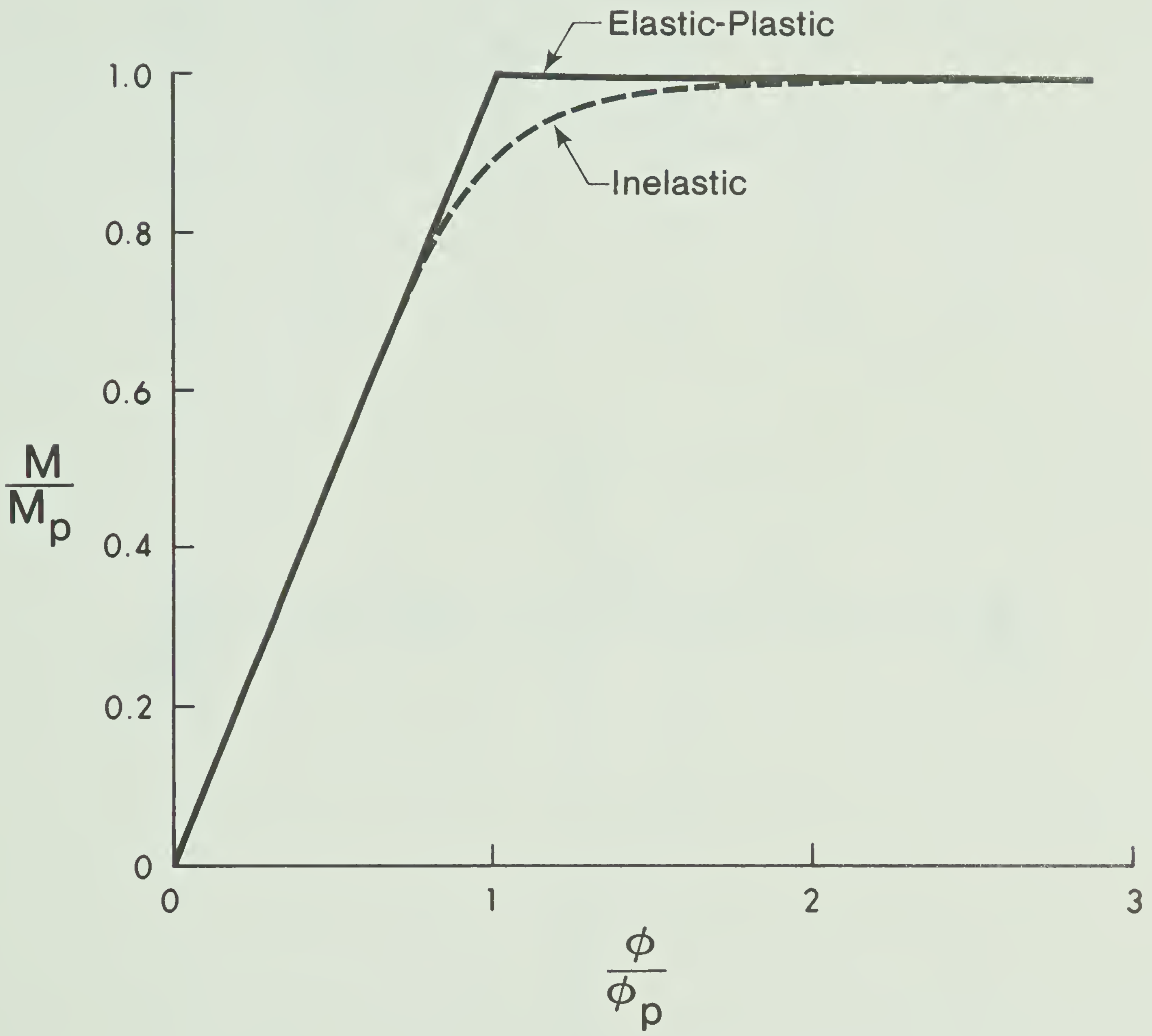
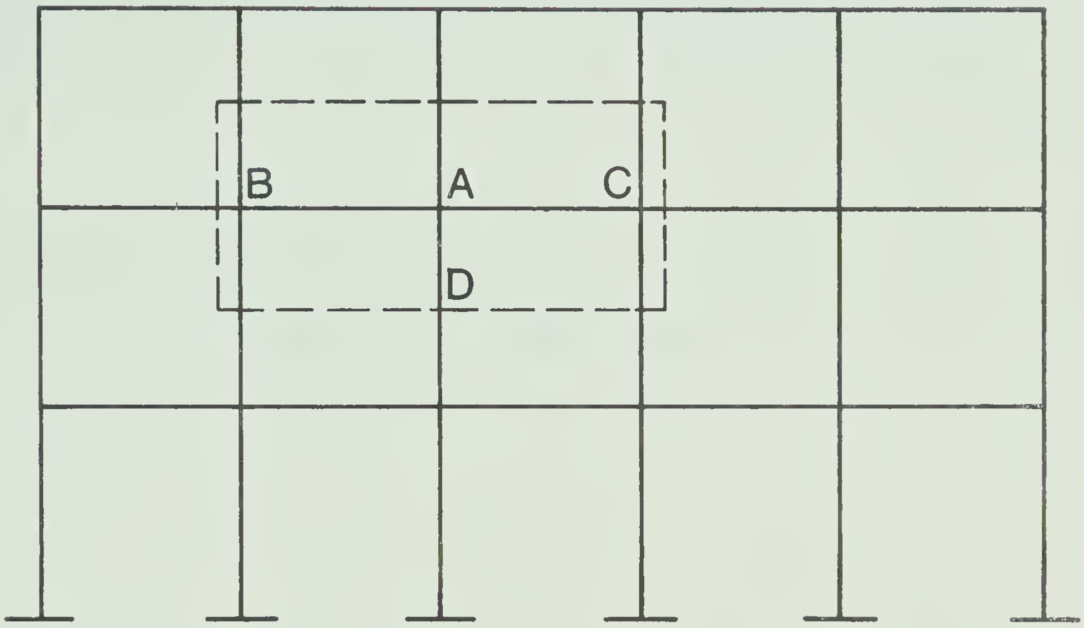


Figure 6.1 Inelastic M-ϕ Response



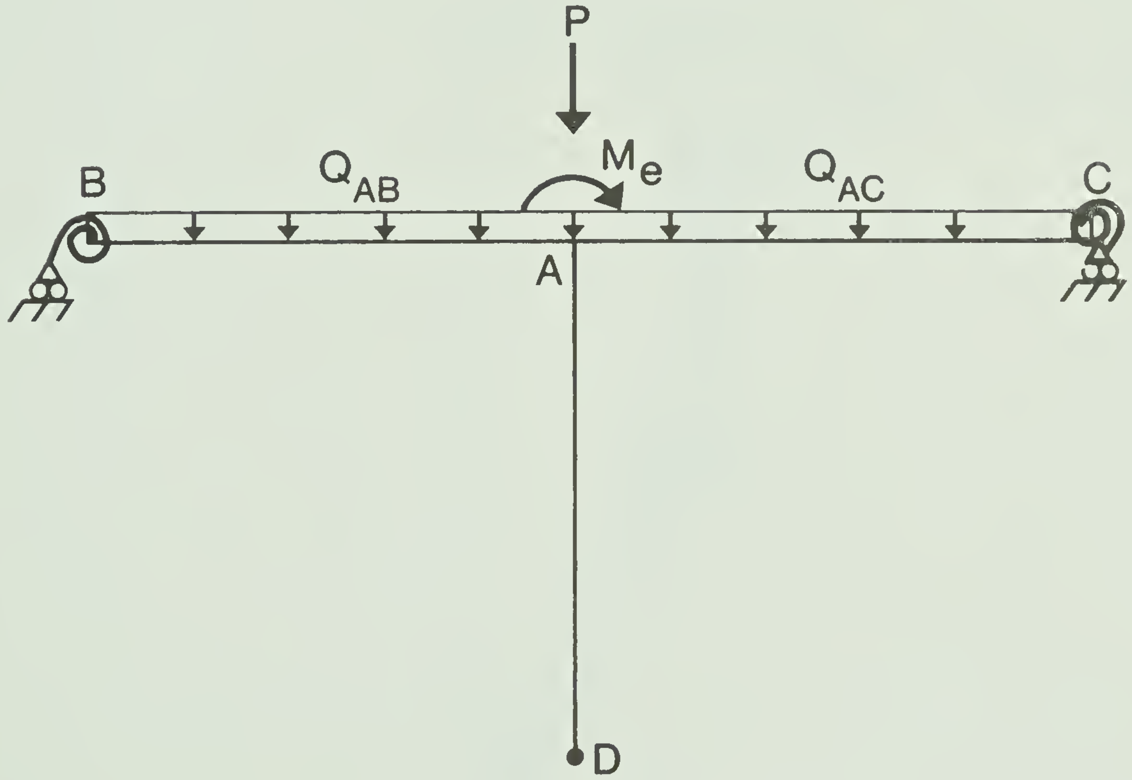


Figure 6.2 Model For Inelastic Analysis

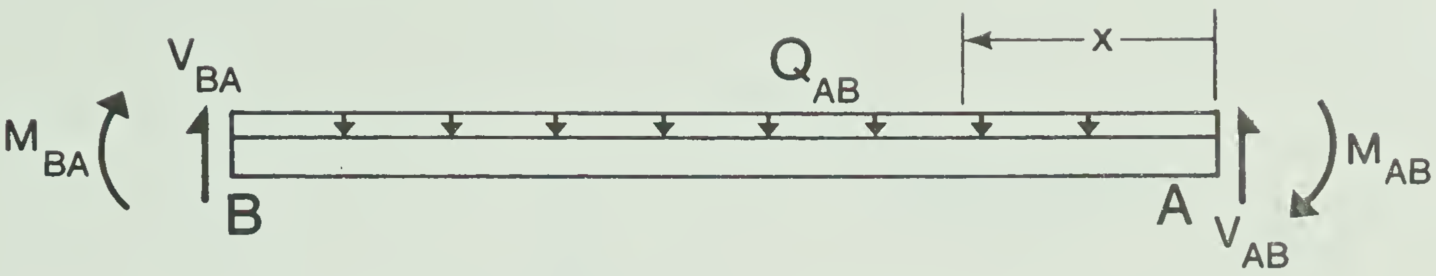


Figure 6.3a Beam Forces

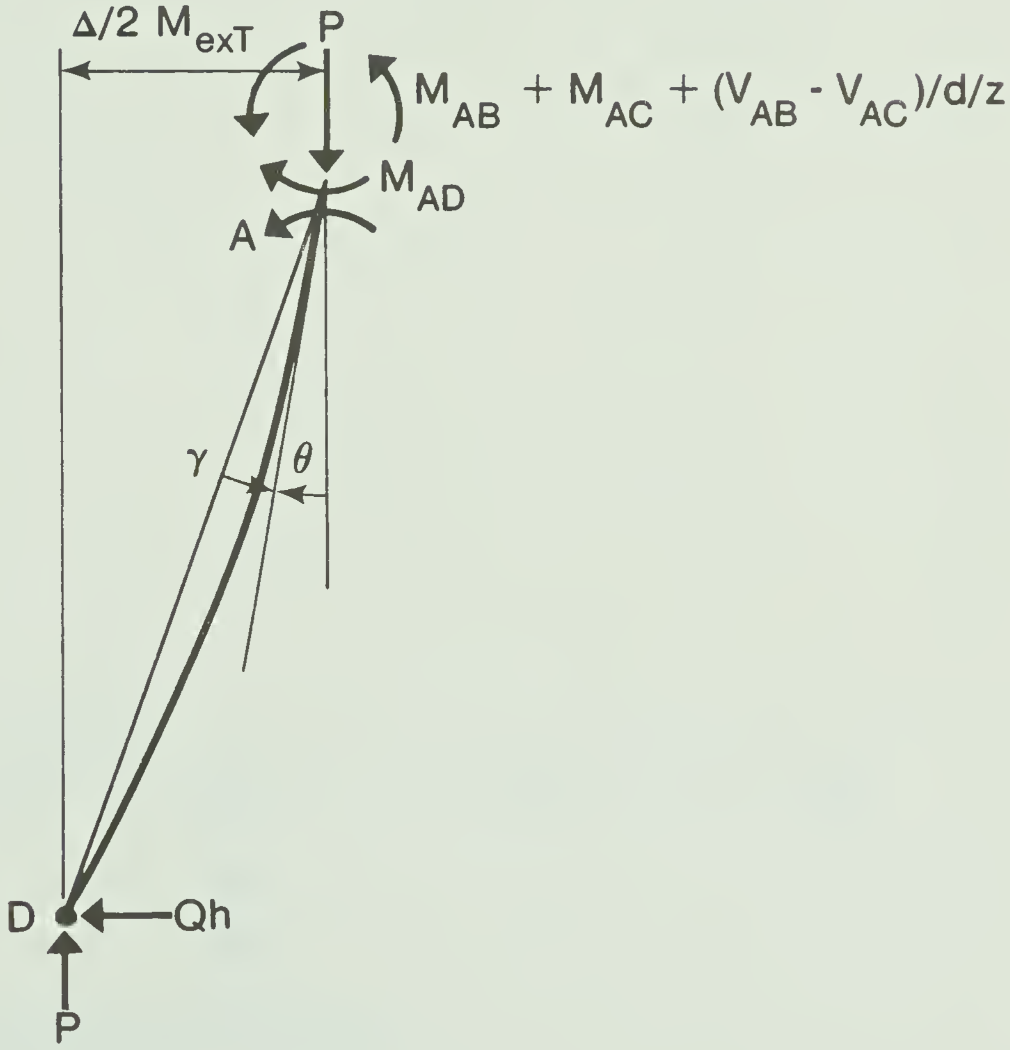


Figure 6.3b Column Forces

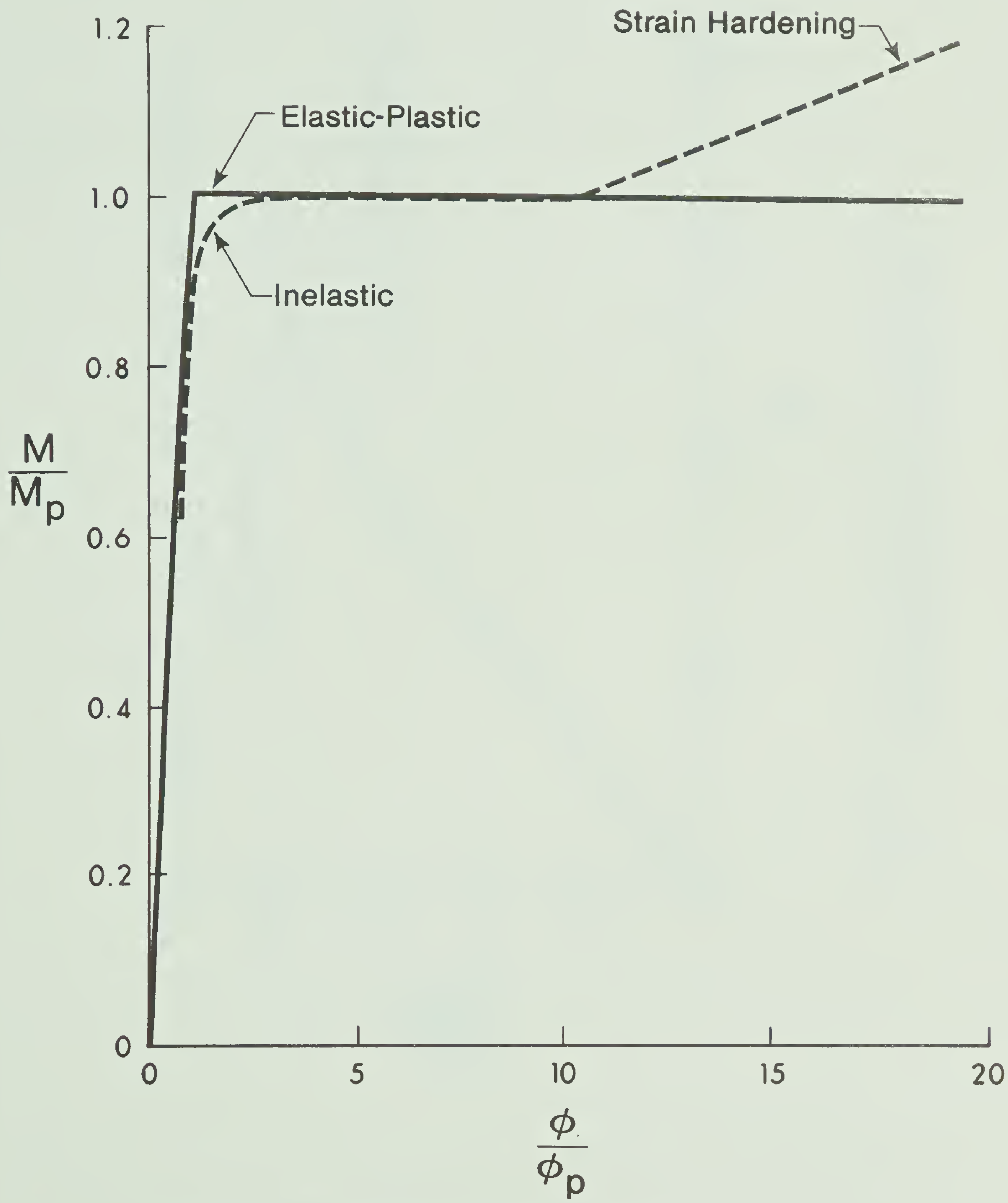


Figure 6.4 Moment-Curvature Relationships

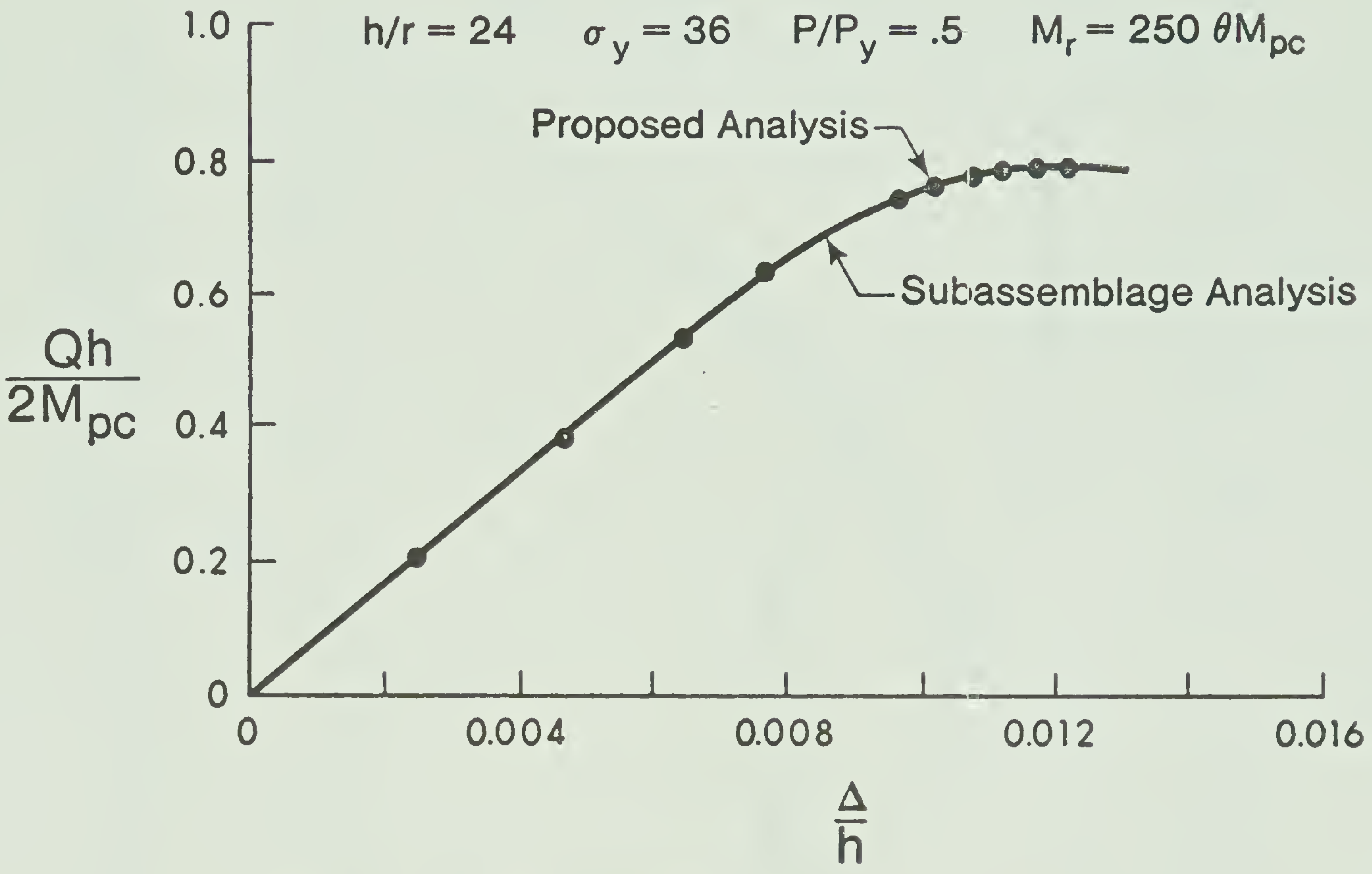


Figure 6.5 Comparison Between Proposed Analysis And Subassemblage Analysis, No Beam Loads

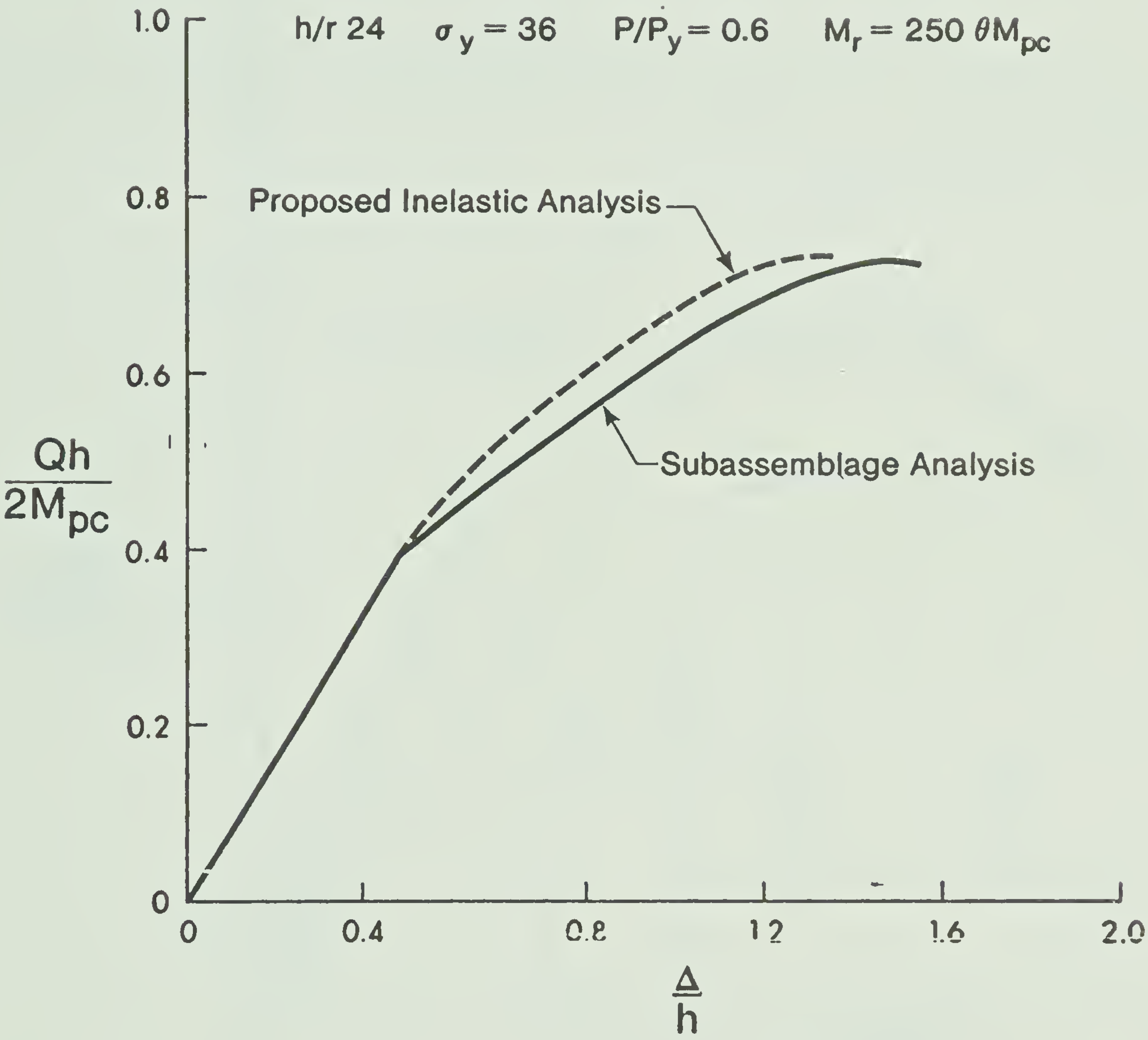


Figure 6.6 Comparison Between Proposed Analysis And Subassemblage Analysis, Beam Loads Included

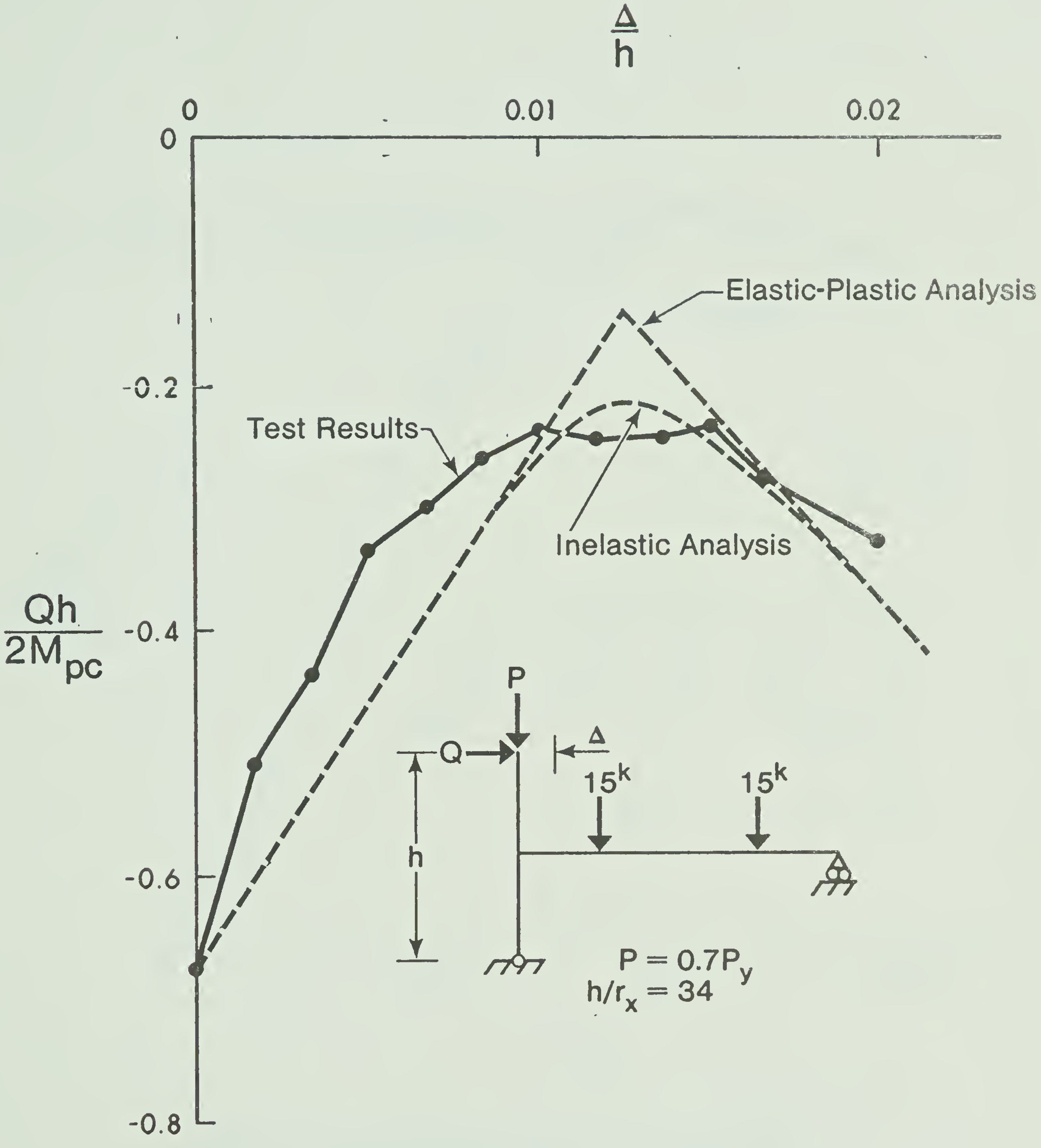


Figure 6.7 Load-Sway Relationships For Frame RC-1

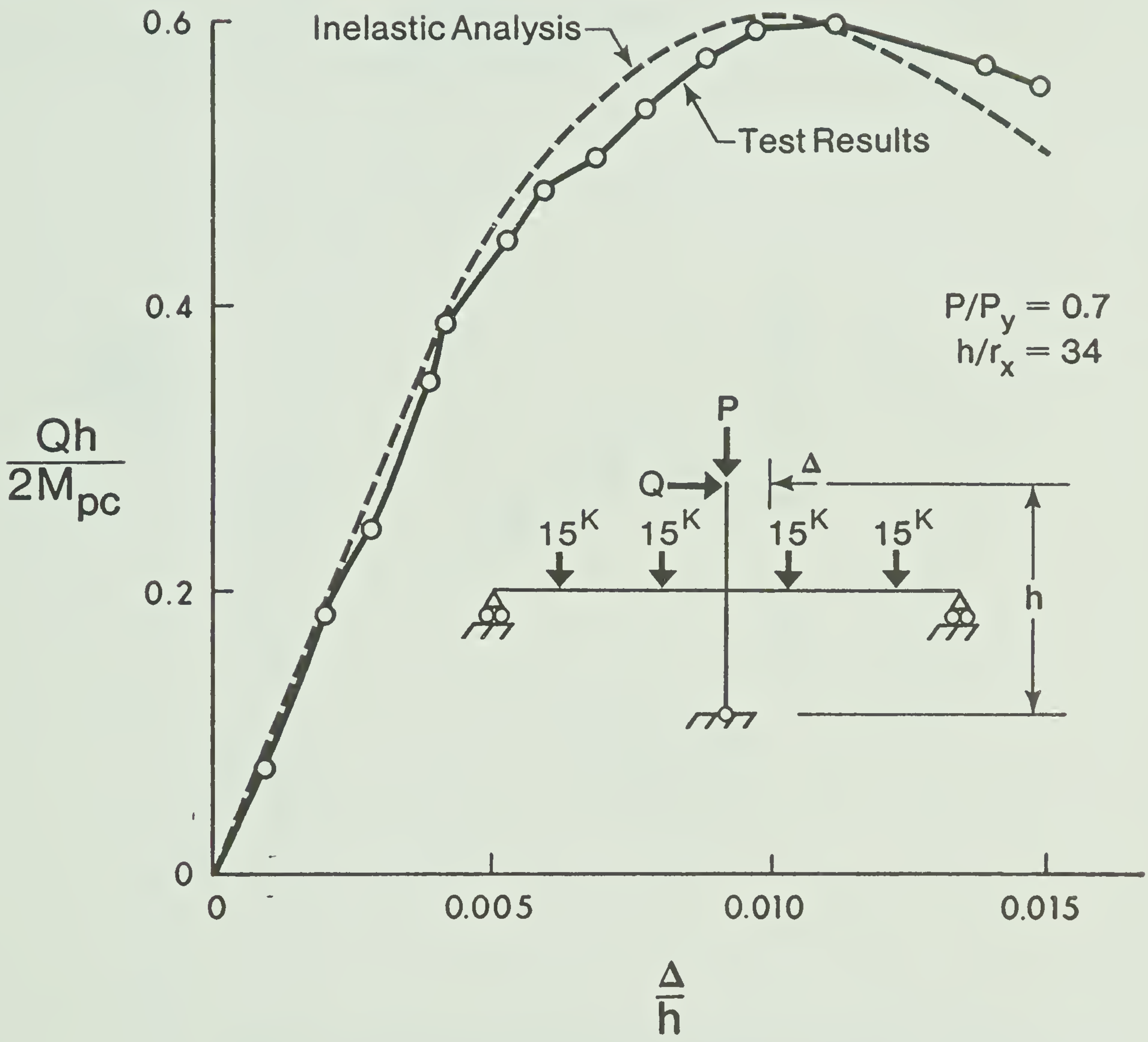


Figure 6.8 Load-Sway Relationship For Frame RC-2

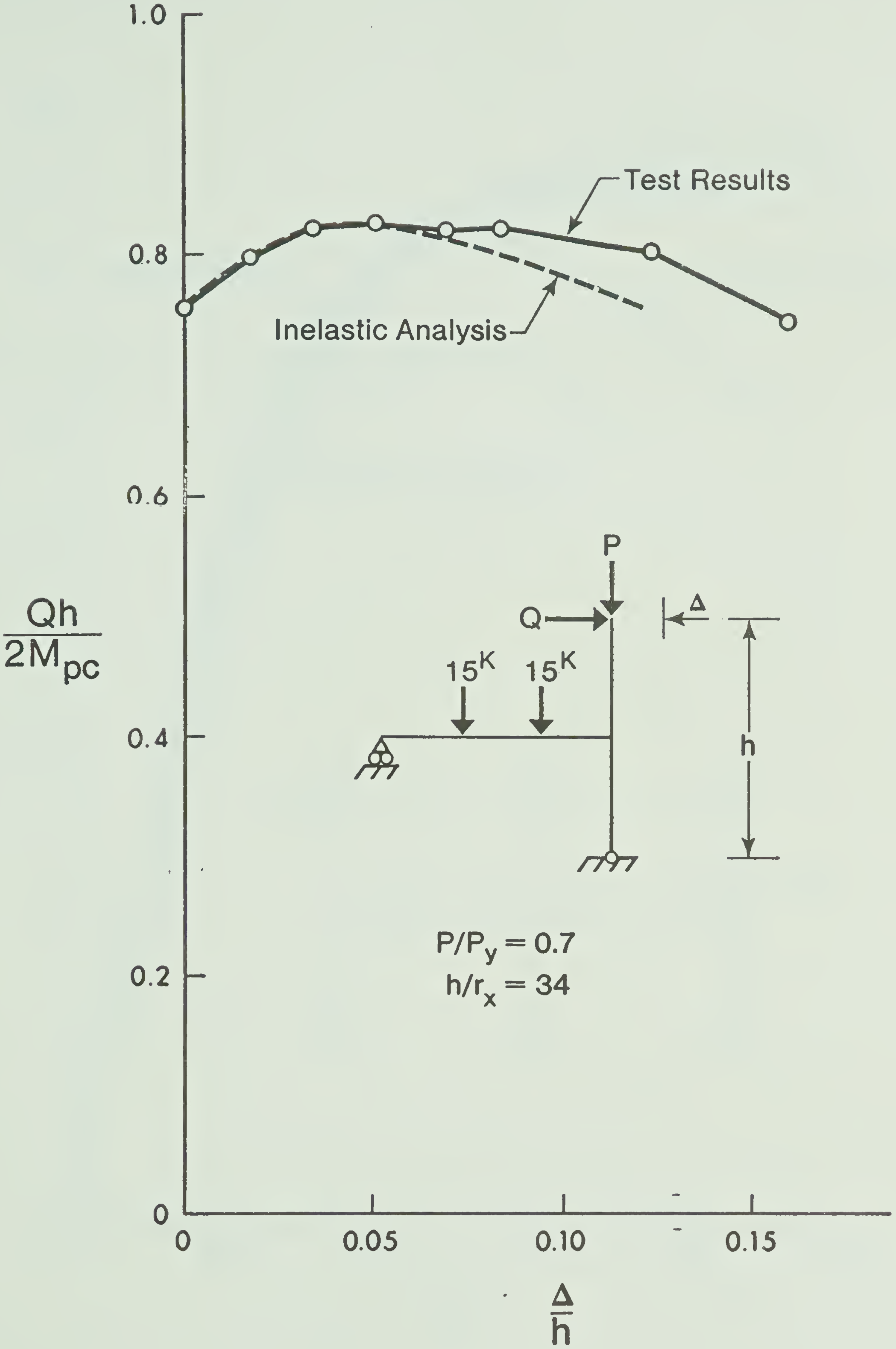


Figure 6.9 Load-Sway Relationships For Frame RC-3

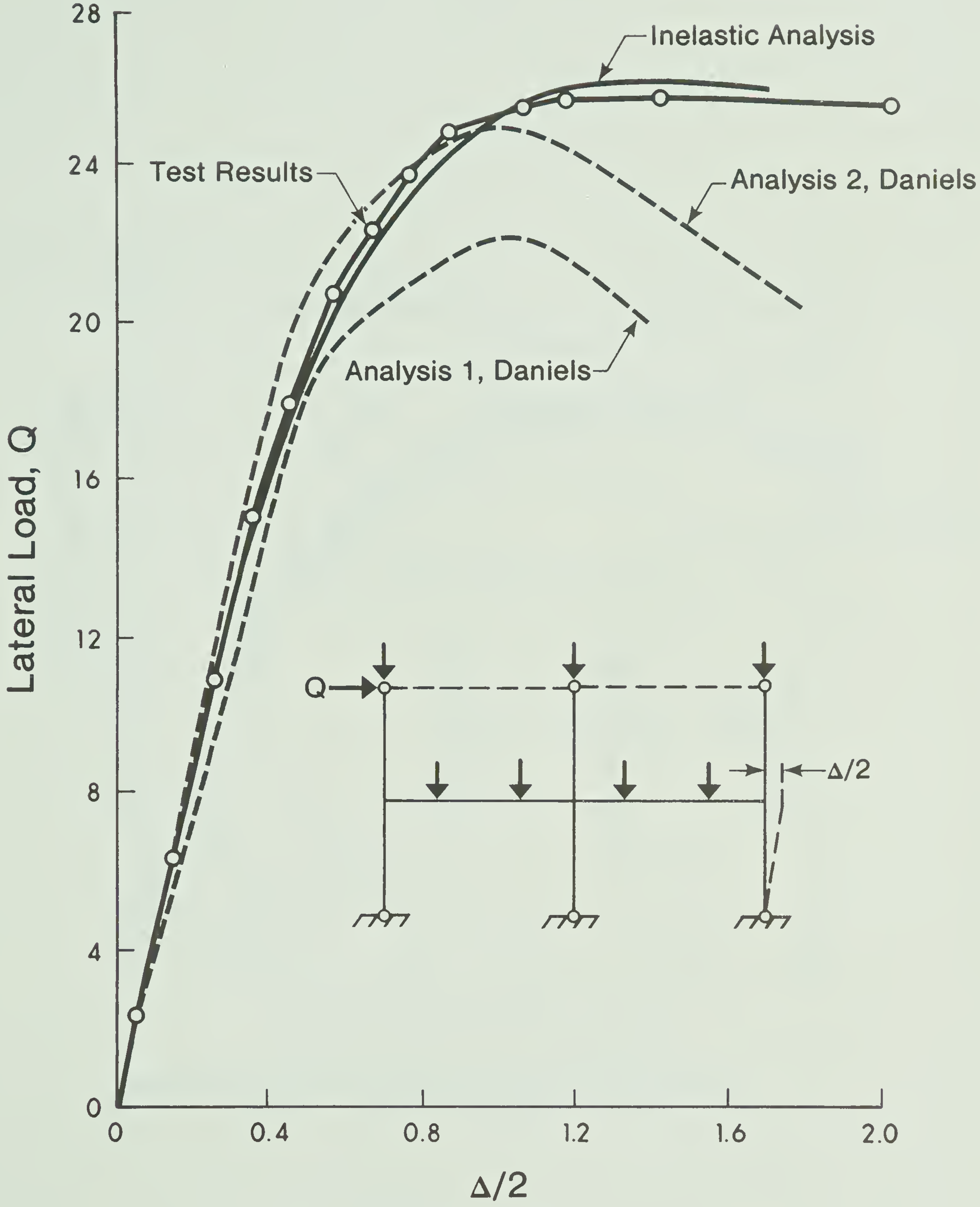


Figure 6.10 Lateral Load Versus Drift Behavior For Example SA-1

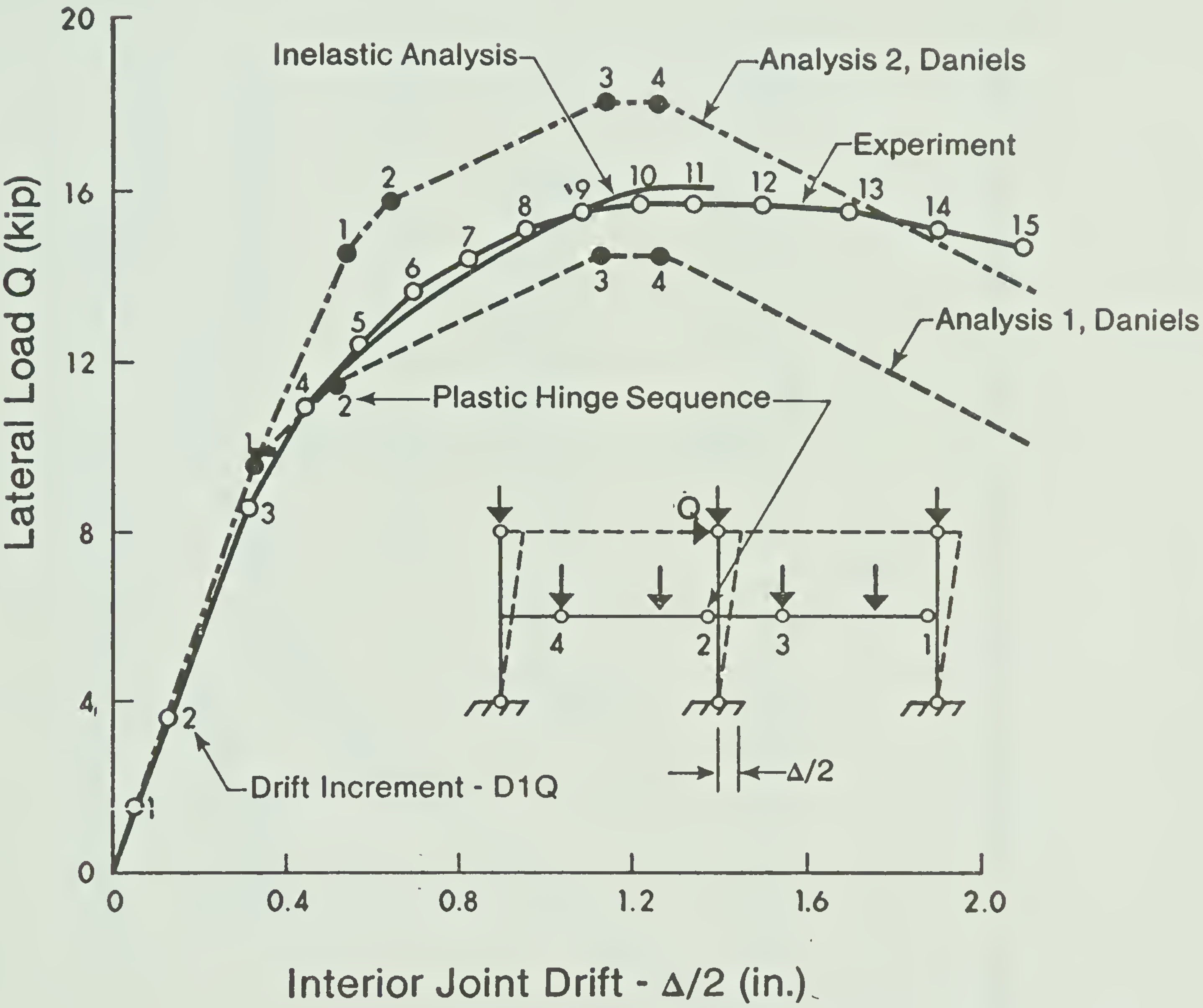


Figure 6.11 Lateral Load Versus Drift Behavior For Example SA-2

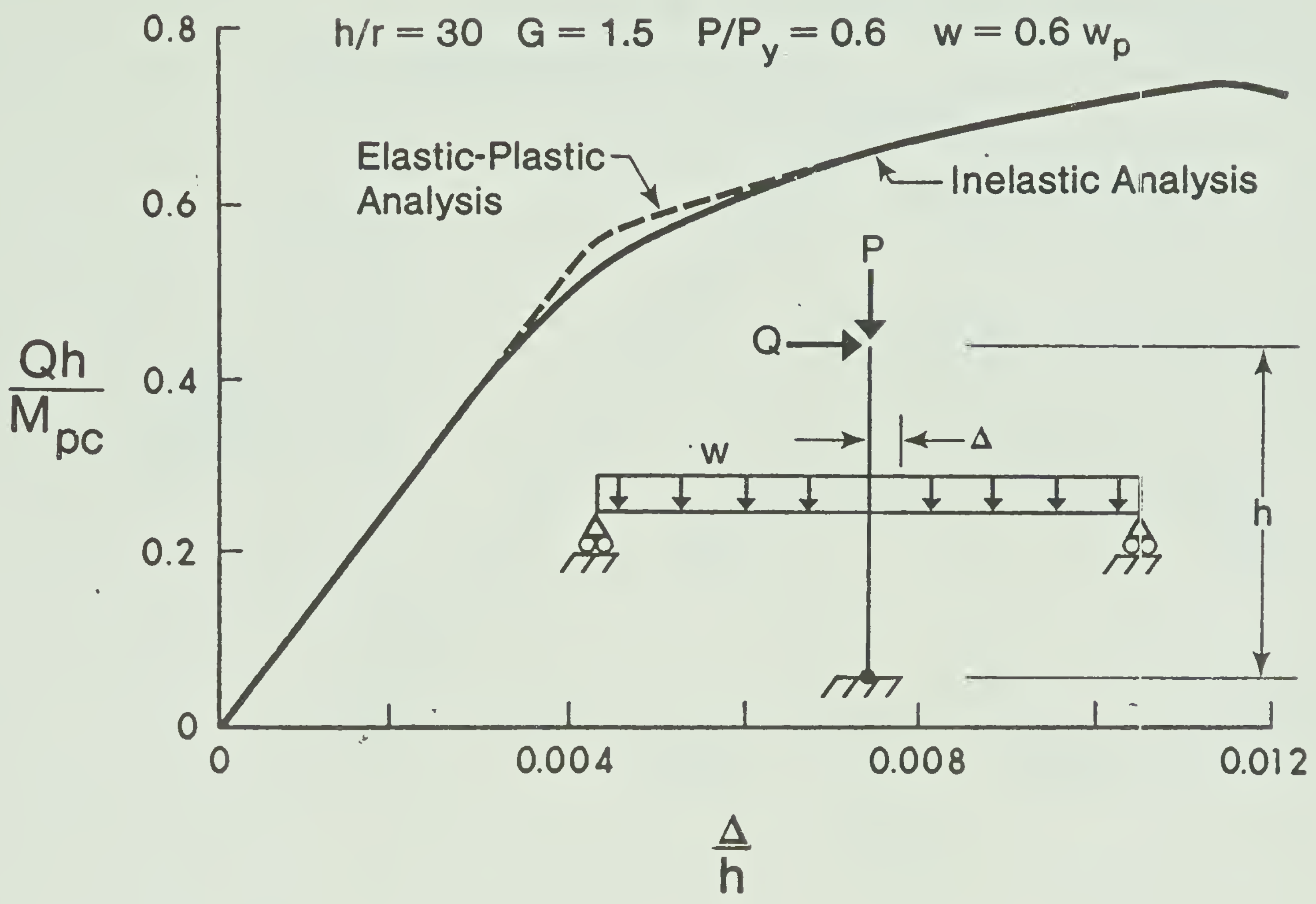


Figure 6.12 Comparison Between Elastic-Plastic And Inelastic Analysis, W = .6Wp

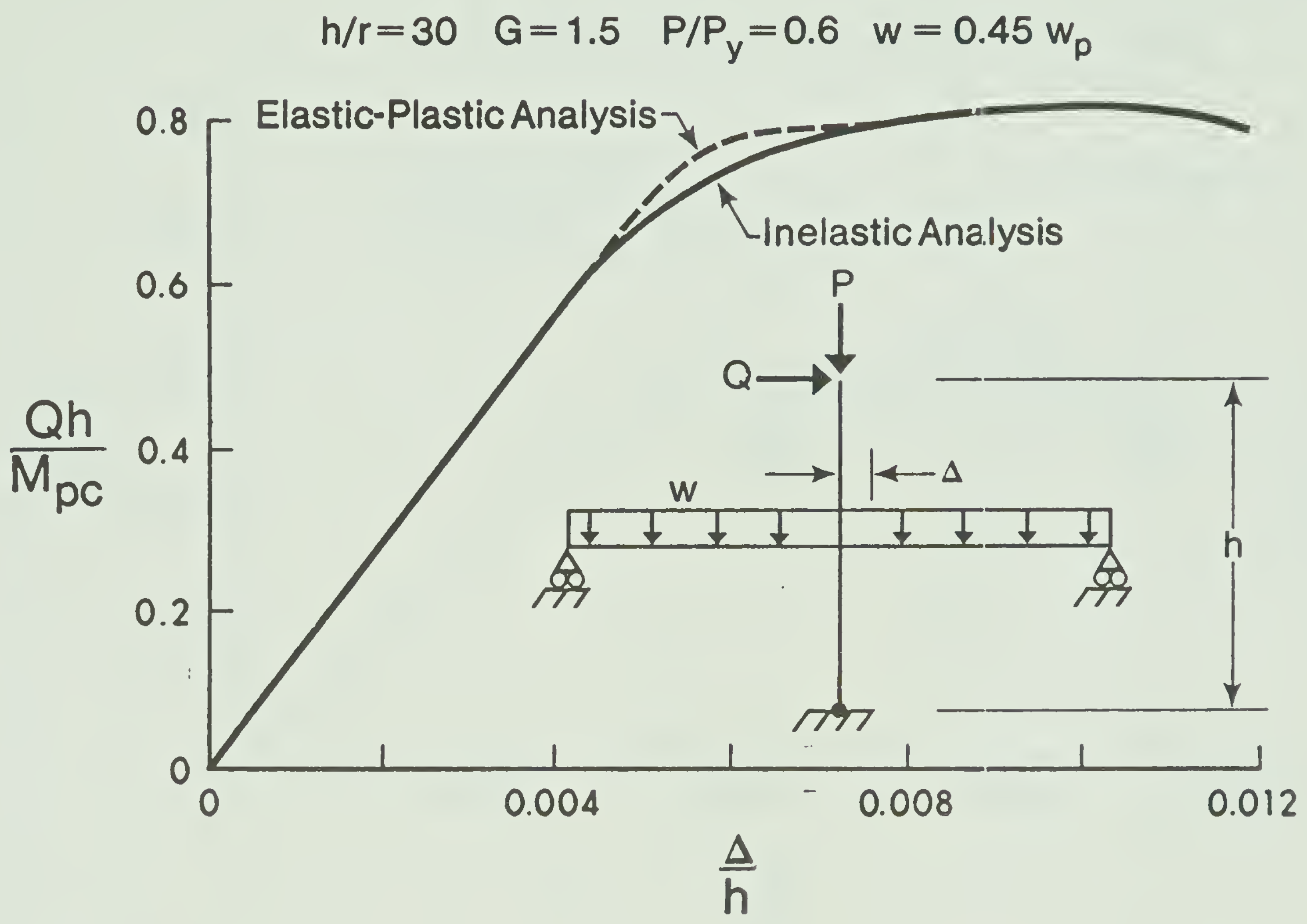


Figure 6.13 Comparison Between Elastic-Plastic And Inelastic Analysis, $W = 0.45W_p$

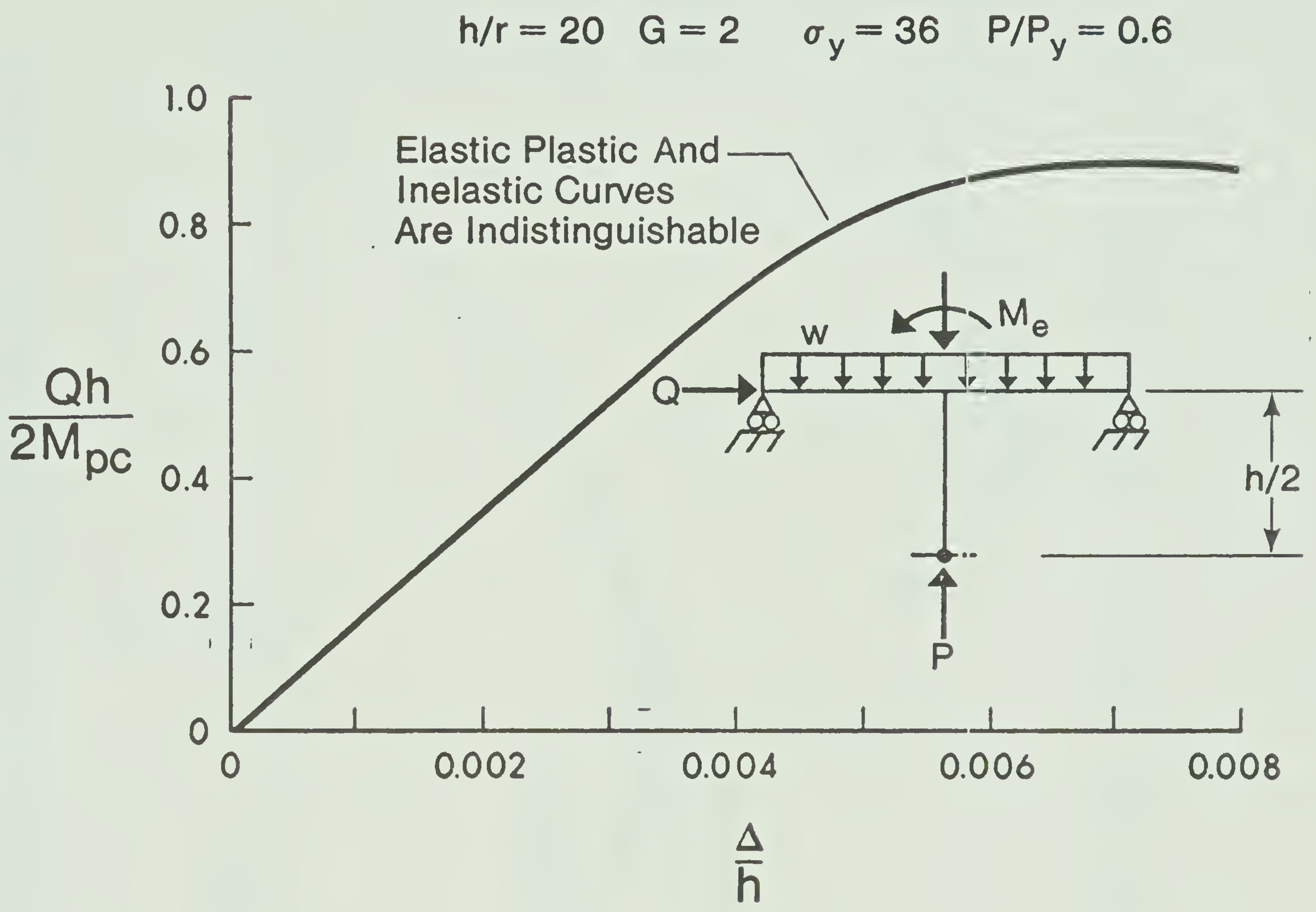


Figure 6.14 Comparison Between Elastic-Plastic And Inelasic Analysis

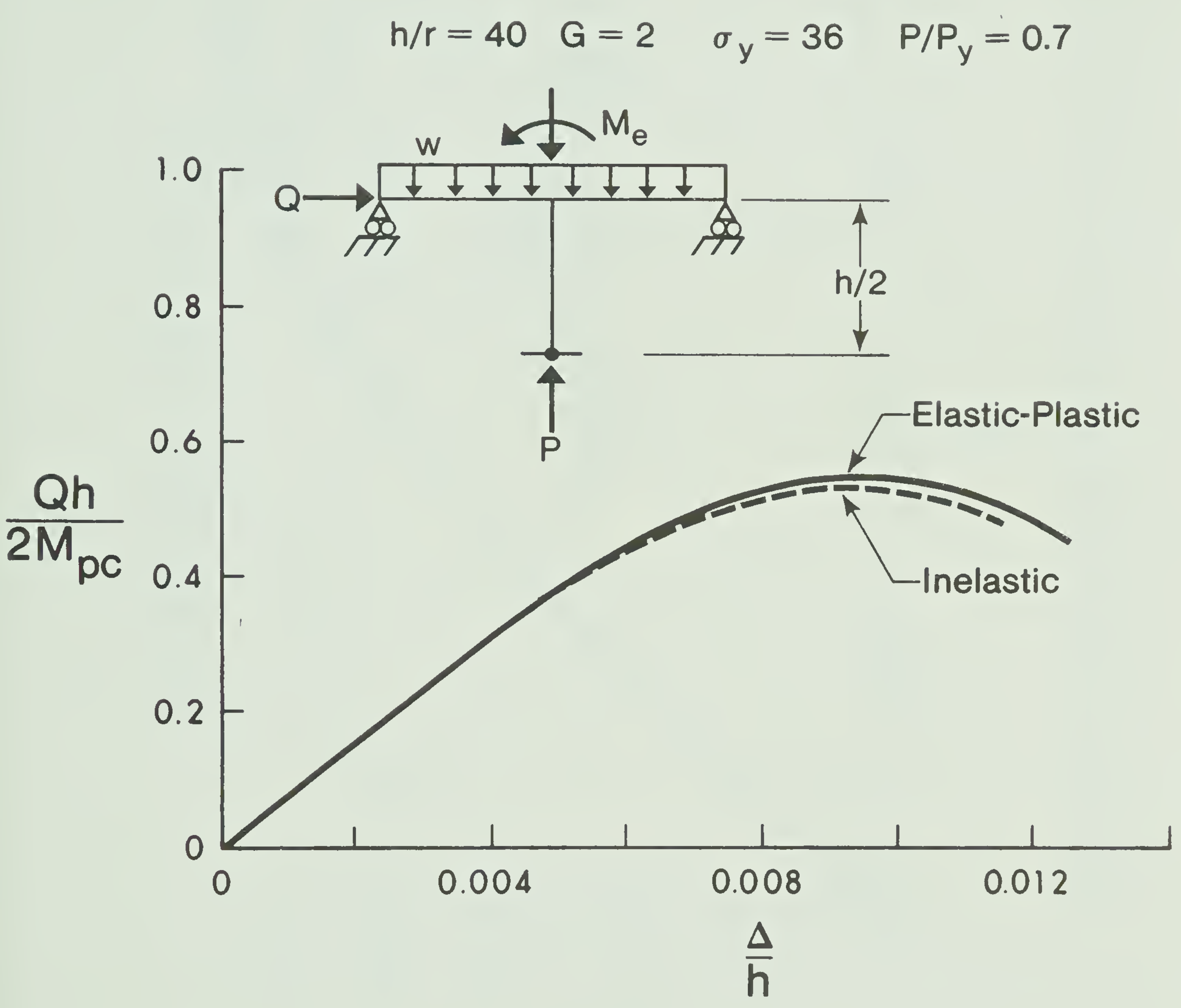


Figure 6.15 Comparison Between Elastic-Plastic and Inelastic Analysis

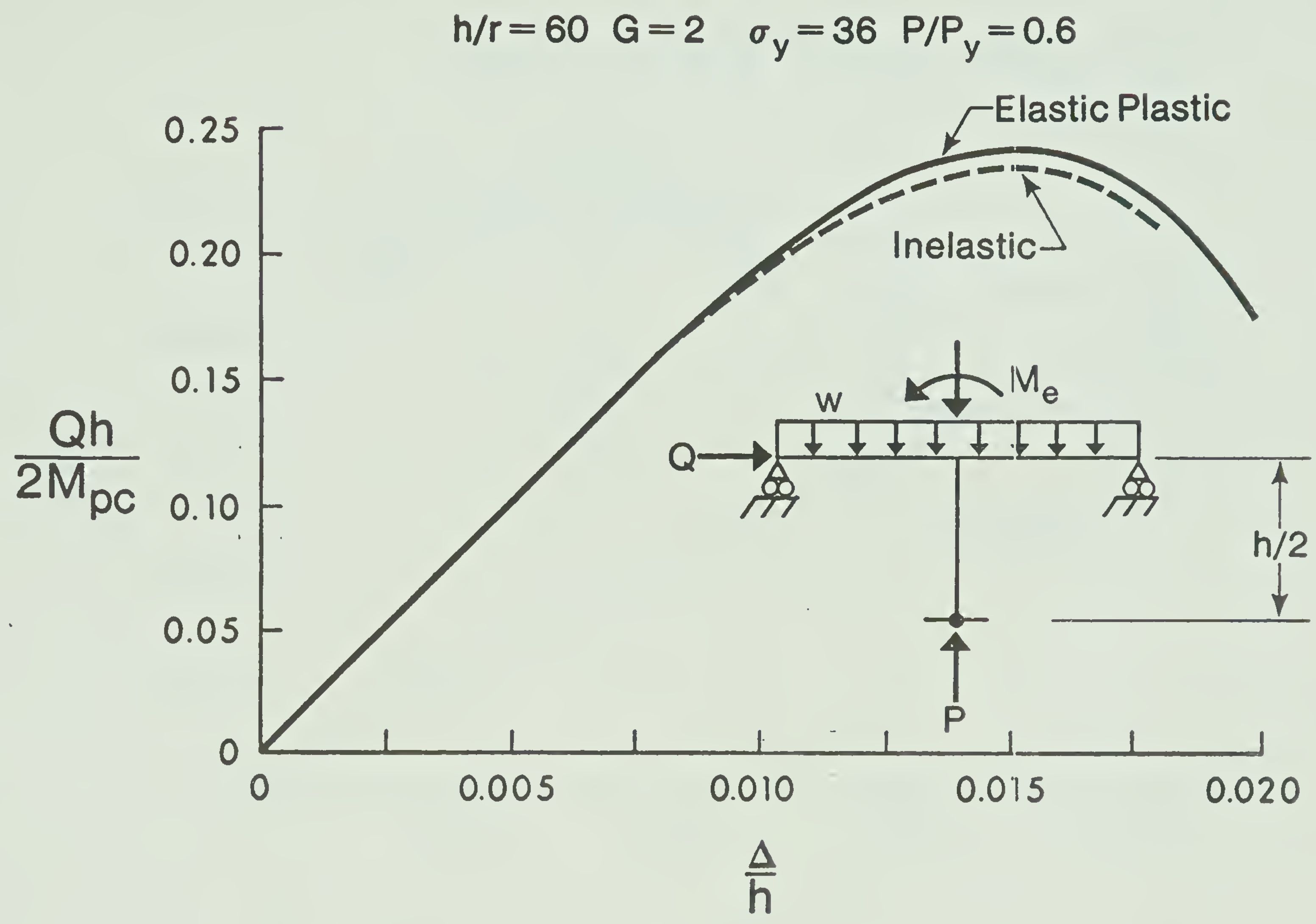


Figure 6.16 Comparison Between Elastic-Plastic and Inelastic Analysis

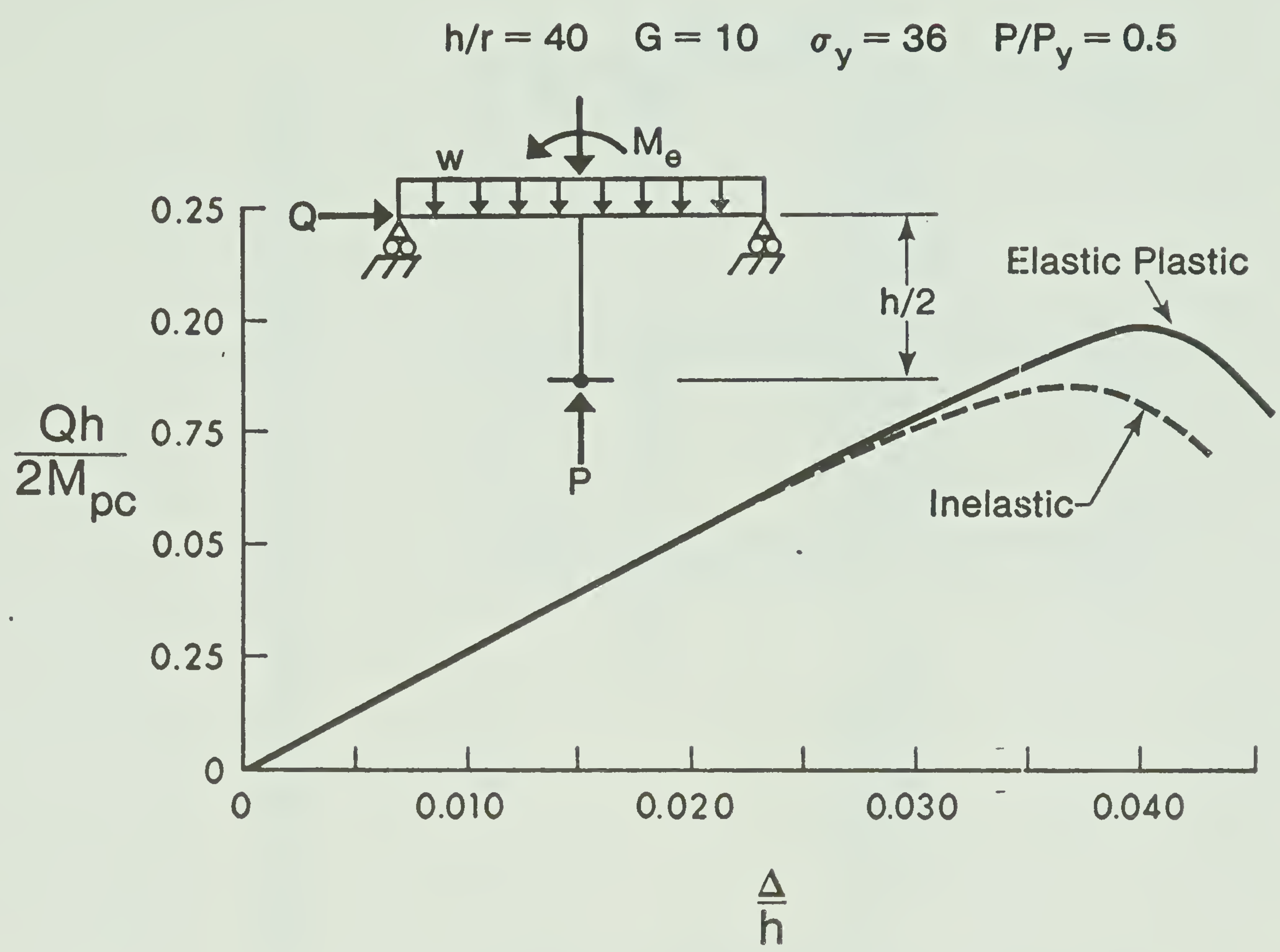


Figure 6.17 Comparison Between Elastic-Plastic and Inelastic Analysis

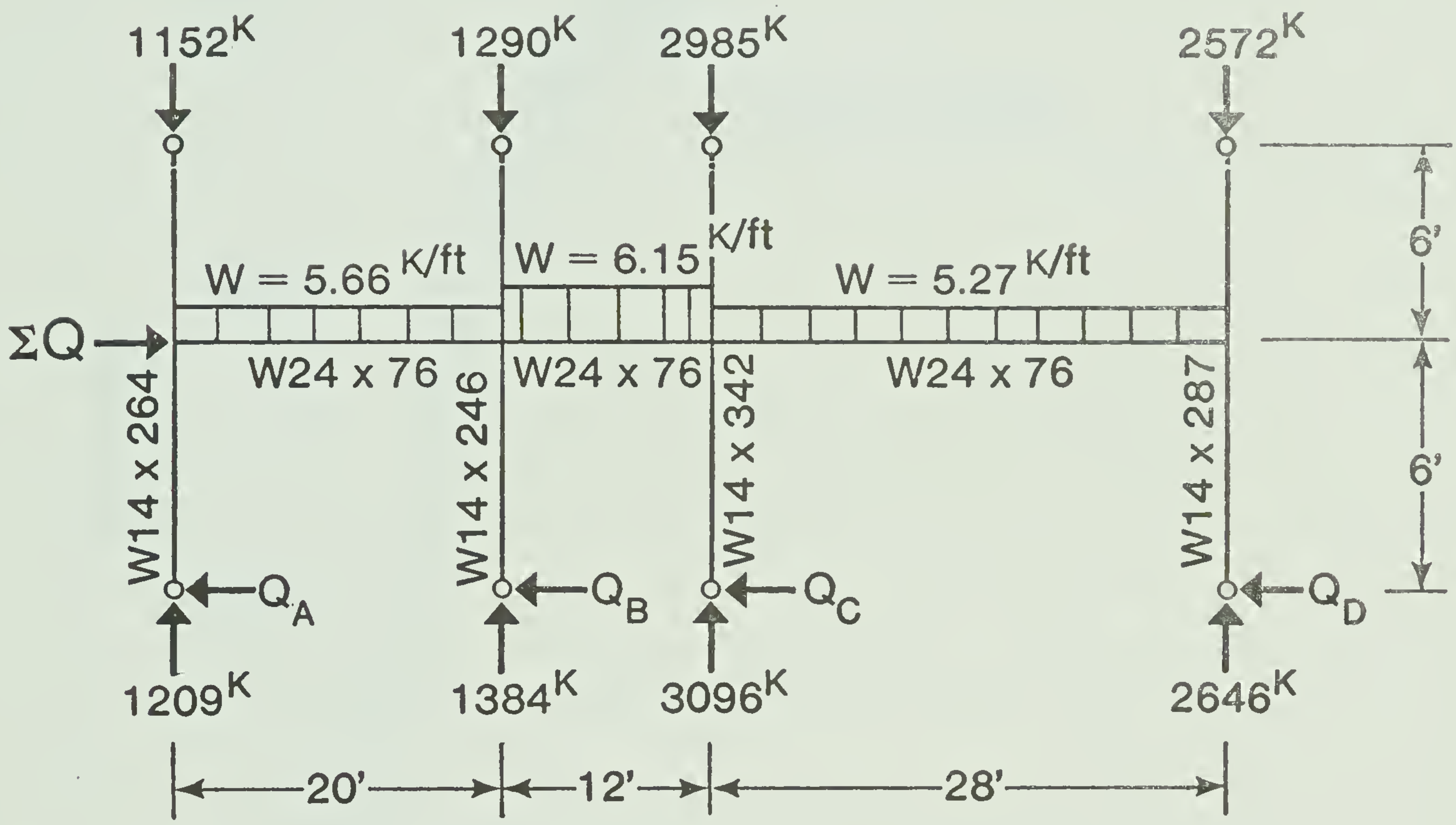


Figure 6.18 Example Frame Analyzed Using Inelastic Analysis

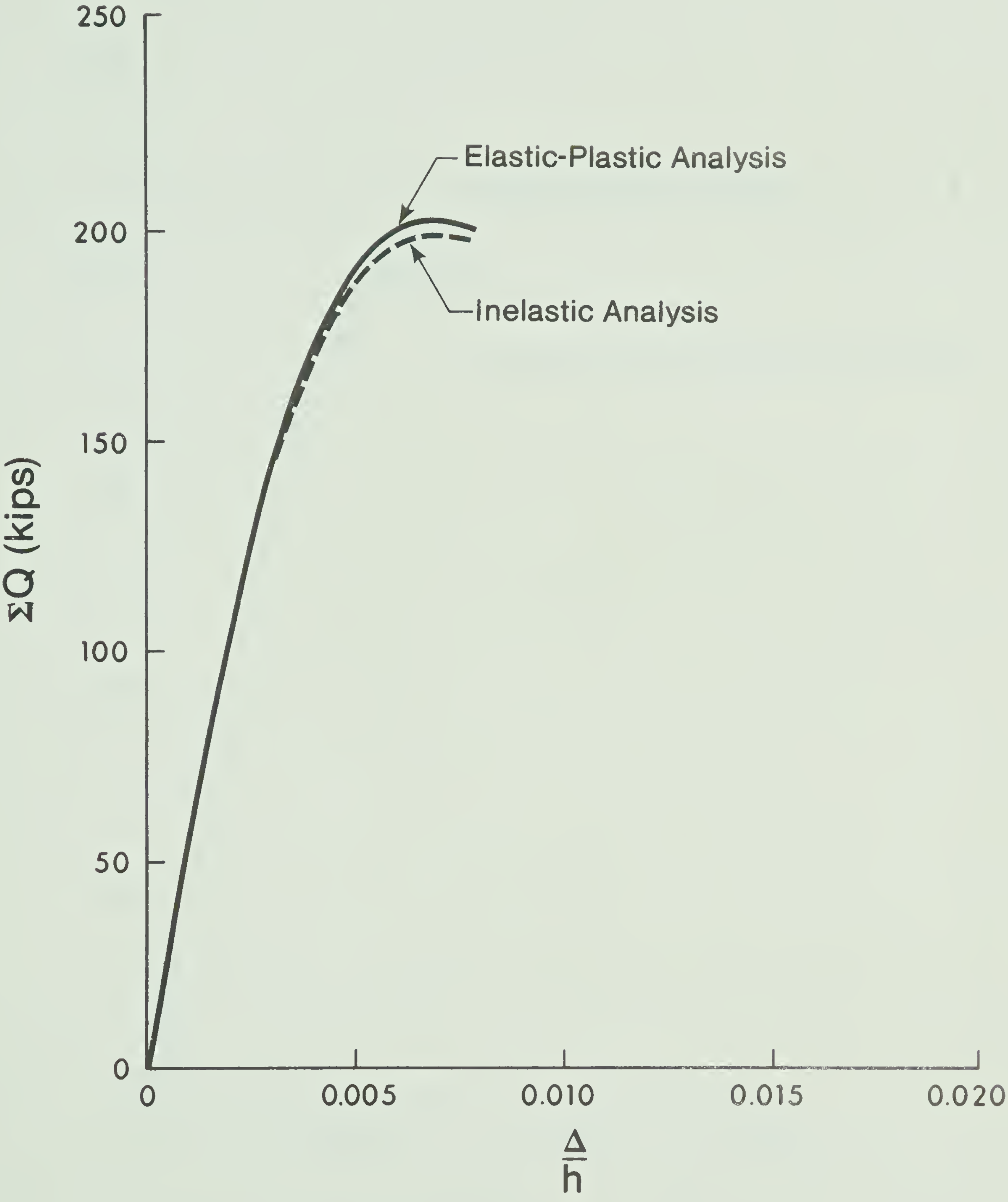


Figure 6.19 Load-Sway Relationships For Elastic-Plastic And Inelastic Analysis Of Example Frame

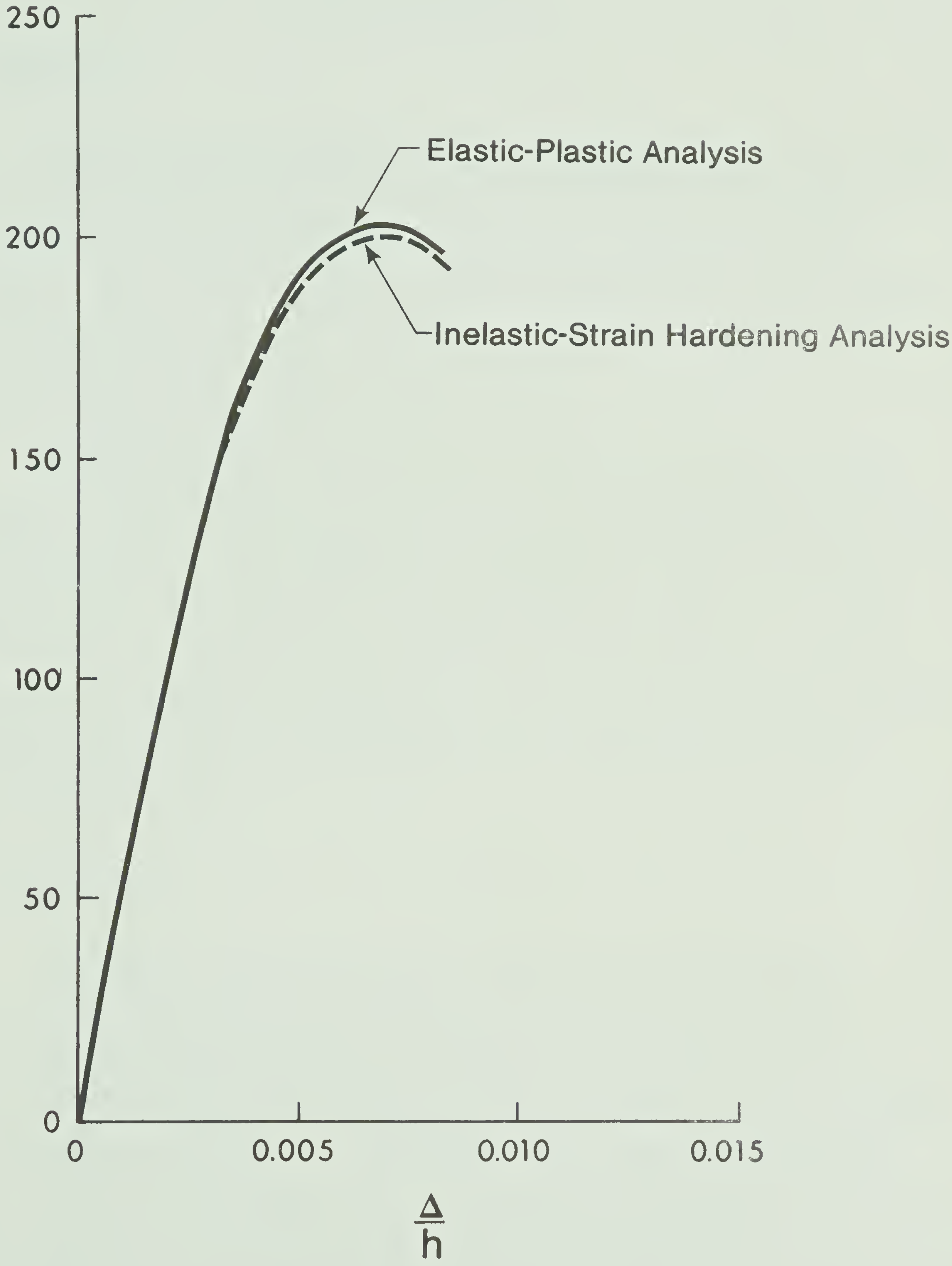


Figure 6.20 Load-Sway Relationships For Elastic-Plastic And Inelastic Strain Hardening Analysis Of Example Frame

CHAPTER VII

DESIGN EXAMPLES

7.1 INTRODUCTION

In the previous chapters, the PΔ procedure for the design of beam columns has been outlined and compared with traditional design procedures. Discrepancies between design assumptions for the PΔ procedure and actual frame behavior have been outlined and limits to the suggested design procedure have been discussed.

In this chapter, an example building frame will be designed using both traditional design procedures and the proposed PΔ procedure. The frame will also be designed using a proposal to neglect PΔ effects altogether (46). In addition, an example of the design of a "leaned frame" will be presented. The behavior of the frames that result from each design procedure will be compared using a second order elastic-plastic analysis (14).

7.2 FRAME DESIGNS

The design loading and frame geometry for this study was selected from a study performed at Lehigh (46). The frame is a 26-storey, three-bay symmetrical frame as shown in Figure 7.1. The design loads are shown in Figure 7.1. The member sizes used for preliminary analyses of the structure were the sizes resulting from Design 3 of the Lehigh Study (46). This particular frame was chosen for this study as it was representative of typical medium to high office buildings that could be expected often in practice rather than an extreme structural

form. A moderate wind load was used in this study as it is believed the influence of P-Delta moments is reduced by higher wind loads (46).

For the purpose of this design, the following assumptions were made:

1. ASTM A36 steel is used for beams and columns.
2. All members are oriented with their web in the plane of the frames and are subjected to strong-axis bending.
3. The frames are braced in the out-of-plane direction at the joints and this bracing system is designed for sway effects. The column ends are effectively pinned for weak axis buckling.
4. Lateral-torsional buckling of the beams is prevented by the floor system.
5. Local buckling of flanges and webs does not occur.
6. Member lengths are defined by their centrelines.
7. The National Building Code of Canada live load reduction formula is used to compute the beam and column loads.
8. Wall and parapet loads are applied as concentrated loads at the exterior joints.
9. Wind loads are applied as concentrated loads at the exterior joints.
10. No composite action occurs between the beams and the floor system.
11. Behavior of frames is assumed to be linear and elastic.

The frames were designed for the effects of combined vertical and lateral loads or vertical loads only whichever produced the more critical effect. In each case, the preliminary analyses were performed using the computer program outlined in Reference 14. The beams were proportioned to resist the moments developed in the preliminary analysis.

The column sizes were chosen by a computer program prepared for this investigation. This program was used to provide a consistency of column size choice for each specified design method and resulting member loads. To ensure consistency, no design over-stress was allowed. The column selection was performed as follows:

1. Design axial loads and end moments were input from the results of the preliminary analysis.
2. Preliminary member sizes were used to calculate effective length factors based on the model specified by the design procedure.
3. A series of prospective column sizes were input along with member properties in tabular form. These prospective columns were input in order of increasing member weight.
4. The suitability of each column, in order of increasing weight, was checked in the beam column interaction equations (Equations 5.6 and 5.7) and Equation 7.1.

$$\frac{Mf}{Mr} \leq 1.0 \tag{7.1}$$

where Mf is the factored column moment

Mr is the factored bending resistance of the

column.

If the column size failed any one of these conditions, this size was rejected and the next member size was tried. The first size to satisfy all conditions was output as the design size for that loading condition.

5. The final column was selected on the basis of the most critical column size for exterior columns and interior columns and interior columns based on a two-storey height of column.

The differences between design procedures used were as follows:

Design A

For Design A, the beams and columns were proportioned using CSA S16.1-1974 (24). The analysis used to obtain factored member loads and forces was a second order elastic analysis that included the effect of axial load on member stiffness and axial shortening of columns. Thus, when checking the suitability of members, ω and K were calculated using the sway prevented models; the ratio R1 was checked for each storey.

Design B

For Design B, the beams and columns were proportioned using CSA S16.1-1974. The factored member loads and moments were obtained from the results of an elastic first order analysis of the preliminary design. Thus, when checking the suitability of column members ω and K were calculated using the sway permitted models.

Design C

To provide a comparison between a frame designed using a P-

Delta limit states design and a frame designed using traditional allowable stress procedures, a frame was designed using CSA S16-1969 (2). The working loads were obtained from the results of a first order analysis of the frame under specified loads. Thus, the column suitability was checked using the sway permitted model to check the column interaction equations as specified in CSA S16-1969 (2).

The resulting member sizes are summarized for each design method in Table 7.1.

7.3 ANALYSES OF DESIGN EXAMPLES

Each of the frames designed as outlined in the previous section were analyzed using an incremental elastic-plastic second order analysis (14). For this analysis, dead loads were multiplied by a dead load factor of 1.25, then held constant. The live loads and wind loads were increased proportionately by increments of loads factor, λ, equal to 0.01.

In the case of vertical loads only, the effect of initial eccentricities were simulated by applying a small lateral shear equivalent to the sum of vertical loads acting on the storey multiplied by the assumed initial deformation, 0.002 in/in. Recent studies have shown this assumption to be overly conservative (47); however, this value was consistent with the allowance presently suggested and with the value of initial imperfection used for the design of frame A (24).

The results of the analyses of the frames subject to combined loading are shown in Figure 7.2. The results of Design A are shown as the solid line, the results of Design B are shown as the dashed line,

and the results of the analysis of Design C are shown as the broken line. In each case, load factor λ, is plotted against frame sway, Δ. As the increments used for λ were 0.01, the apparent accuracy of critical points on these curves is approximately 1%.

For Design A, the first hinge occurred at $\lambda = 1.18$. The progression of hinges to failure is shown in Figure 7.3. At ultimate load, $\lambda = 1.33$, (specified by a negative value for the determinate of the stiffness matrix), there were 39 beam hinges and 51 column hinges. At specified load, $\lambda = 0.7$, the overall sway index = 0.00237.

For Design B, the first hinge occurred at $\lambda = 1.07$. The progression of hinges to failure is shown in Figure 7.4. At ultimate load, $\lambda u = 1.34$, there were 71 beam hinges and 15 column hinges. At specified loads, $\lambda = .7$, the overall sway index = 0.0025.

For Design C, the first hinge occurred at $\lambda = 1.17$. The progression of hinges to failure is shown in Figure 7.5. At ultimate load, $\lambda = 1.46$, there were 57 beam hinges and 28 column hinges. At working loads for allowable stress design, $\lambda = 0.75$, the overall sway index = 0.00244.

The results of the analyses of the example frames under vertical loads only are shown in Figure 7.6. The results of the analysis of Design A are shown as the solid curve. The first hinge occurs at $\lambda = 1.81$, while ultimate load is reached at $\lambda = 2.12$ due to yielding in critical columns in the 21st storey. The results of the analysis of Design B are shown as the dashed line in Figure 7.6. The first hinge occurs at $\lambda = 2.09$ and the ultimate load at $\lambda = 2.49$ due to yielding of critical columns in the 17th storey. The results of the

analysis of Design C are shown as the broken line in Figure 7.6. The first hinge occurred at $\lambda = 2.26$ with failure due to yielding in the top stories at $\lambda = 2.74$.

7.4 DISCUSSION OF DESIGN EXAMPLES

7.4.1 COMPARISON OF DESIGN A AND DESIGN B

Since Design A is an example of a design using the $P\Delta$ technique and Design B is an example of the traditional allowance for sway effects, a comparison of these examples is a direct comparison of these two design techniques. A comparison of the member sizes shows that the inclusion of second order effects in the analysis results in larger sizes for the beams as the $P\Delta$ procedure includes the second order moments in design of the beams. The more rational approach to design of columns in the $P\Delta$ technique leads to generally smaller column sizes. The estimated weight of Design A is 395,900 pounds which is 4% less than the 411,300 pound weight of Design B.

For the $P\Delta$ design procedure, the column sizes were found to be governed by the strength interaction equation, Equation (3.20). The sway effects ratio, $R1$, was found to vary between 0.09 and 0.12, well within the limits suggested in Chapter 5. The column sizes in the traditional design procedure were mainly governed by the stability interaction equation, Equation (3.21).

Since the analysis used to study the behavior of the frames does not include out-of-plane effects, there was some question as to the validity of the assumption that members were only laterally braced at storey levels. This assumption is certainly more realistic when the

behavior of real frames are considered. To check this point, a design of each frame was performed using the assumption that the frame is completely braced in the out-of-plane direction. The resulting member sizes were exactly the same as found when members were only braced at storey levels. For the PΔ design, since the member sizes were controlled by member strength, there was no effect if the members were braced at storey levels. In the case of the traditional design, it was found that $K_x L/r_x$ was always greater than $K_y L/r_y$ for these larger member sizes, hence, this assumption had no effect on the design.

A comparison of the results of the analysis of each frame shows that Design A is approximately 5% stiffer at working loads. Design A developes a plastic hinge at $\lambda = 1.18$. For limit states design, the margin of safety for combined live loads is 0.70 times 1.5. There is, however, an additional allowance to account for material performance of 1/0.9. Thus, the resulting factor of safety for combined loading is 1.17. Design A satisfies this factor of safety.

For Design B, however, the first hinge is formed at $\lambda = 1.07$ which is less than the specified safety factor 1.17. This premature hinge formation results from neglect of the second order moments in the design of the beams. While a redistribution of moments in the frame allows a continued increase in load carrying capacity of Design B beyond $\lambda = 1.17$, there is no allowance for this redistribution of moments unless beams are designated as Class 1 sections and are properly braced.

A comparison of the frames at ultimate load shows that the frames fail at nearly the same load, but in somewhat different manners. Design A develops considerably more column hinges while the failure of

Design B is essentially related to the formation of beam hinges. As a result, Design B exhibits more overall lateral sway at failure and thus, tends to absorb more energy before collapse.

Thus, a design using the PΔ technique may tend to produce a strong beam, weak column structure; while the traditional technique tends to produce a weak beam, strong column structure. The tendency for traditionally designed frames to be somewhat more flexible can lead to design being controlled by deflection criteria. Thus, the beam sizes may have to be increased to meet frame stiffness requirements and modify the failure pattern.

A comparison of the results of the vertical loads only analysis shows that design has been controlled by the combined loading case and neither frame is critical for vertical loads. The ultimate load for Design B is somewhat higher than the ultimate load of Design A as the behavior under vertical load is more controlled by column size.

7.4.2 DISCUSSION OF DESIGN C

Since allowable stress design procedures used the traditional method exclusively to allow for sway effects, Design C was included in this dissertation to compare the results of traditional allowable stress design to a limit states PΔ design. The estimated weight of Design C is 424,200 pounds or 7% more than Design A.

Design C is approximately 4% stiffer than Design A at working loads. The first hinge occurred at $\lambda = 1.17$, essentially the same as Design A. Design C, however, has been designed for an effective factor of safety, of S.F. = 0.75/0.60 = 1.25. Thus, Design C does not meet the

factor of safety specified for the design method. This apparent inconsistency results from neglect of the second order moments for the design of the beams.

The ultimate load for Design C is slightly higher than Design A (9%). In addition, the failure of Design C was more ductile with failure related to beam hinges.

The comparison between designs resulting from a limit states design using the PΔ technique and a traditional allowable stress design shows that each design has essentially the same load factor at the limit state, while the limit states design shows an appreciable material savings. The allowable stress design does result in a more favourable collapse condition and considerably more reserve in load capacity and energy absorption.

7.5 FRAMES DESIGNED WITHOUT CONSIDERING FRAME STABILITY

A further design procedure has been presented in the Column Research Council Guide based on research at Lehigh into the behavior of several building frames designed using various methods (31,46). For this design procedure, it is suggested that a frame can be designed using the results of a first order analysis, but with K = 1.0 and $\omega = 0.6 - 0.4\ M_{f1}/M_{f2} \geq 0.4$ if the following conditions are met:

1. The axial loads in the columns are such that C_f/C_r and $C_f/\phi AFy$ are both less than 0.75.
2. The maximum in-plane slenderness ratio does not exceed 35.
3. The bare frame first order sway index is controlled by

$$\Delta/H<(1/7)\Sigma V/\Sigma P$$

where:

H = storey height

Δ = drift of storey due to ΣV

ΣV = total storey shear

ΣP = total gravity load on the storey.

In the Lehigh Study (46), seven frames, varying from a ten-storey, five-bay to a forty-storey, two-bay, were designed using the $P\Delta$ procedure, the traditional effective length procedure, and using the method outlined above. The frames designed using these methods were then analyzed under various loading procedures using a second order elastic-plastic analysis. Based on the results of these analyses, the authors recommended that frames may be designed without consideration of second order effects as long as the conditions outlined earlier in this section were met.

Unfortunately, there were some serious oversights in the study. In designing the frame using the $P\Delta$ procedure, the beams were apparently proportioned using first order moments, thus, the resulting beam sizes were identical for each design. When the $P\Delta$ procedure is properly applied, the resulting beams will be larger, and thus, the frame stiffness will likely increase.

When the design procedure proposed by the authors was applied, it was assumed that columns were completely braced out-of-plane, thus, $K_x L/r_x$, governed the column size. This was not consistent with the assumptions made for the other design methods. For the other designs, the columns were assumed to be braced only at the storey levels in the

out-of-plane direction. The authors were effectively comparing frames designed under different support conditions making comparisons unrealistic.

Finally, the authors were more concerned with a comparison of behavior at ultimate strength of the frames, whereas, for both allowable stress design and limit states design procedures, the limit state for the structure is reached when any member reaches a critical stress or ultimate strength. Therefore, the load factor for the formation of the first hinge is extremely important. In the study, there was little variation of the load factor at first hinge between the different designs. This, however, reflects the fact that the beams were the same for all designs and initial hinges were formed in the beams. When the PΔ procedure is properly applied, larger beams will result and hence, the hinges will form later, as found in Design A.

To further evaluate the Lehigh design proposal, the example frame was also designed using a first order analysis, but with $K = 1.0$ in both directions and $\omega = 0.6 - 0.4\, M_{f1}/M_{f2} \geq 0.4$. The ratio $C_f/\phi AFy$ was checked for each column. In addition, the column slenderness ratios and the sway indices were also checked against the suggested limits.

The design of the frame proved to be quite difficult in this case. The original proposal was made for an allowable stress procedure, while for this dissertation, stability design using Limit States Design was evaluated. Thus, the suggested ratio for axial loads may not be directly applicable and should possibly be modified. In the original proposal, there was no suggested action for the cases when this ratio exceeded. In the design example, if the member sizes were increased

until the ratio was satisfied, the resulting column sizes were considerably larger than the previous examples. For this design example, the ratio was not considered for member selection, but was calculated for discussion.

As in the previous examples, the frame was considered braced laterally at storey levels. For this design procedure, this assumption will affect final member sizes, however, to be consistent with the other designs and normal practice, the same assumption was made. The resulting member sizes are shown in Table 7.1. In this Table, the design method has been designated as Design D. The axial load ratio for the critical members varied between 0.77 and 0.85 for combined loading with values as high as 0.92 for the case of axial loads only. The column in-plane slenderness ratio limit and sway index limit were easily satisfied.

In most cases, the resulting column sizes were the same as the sizes for Design A. A number of the exterior columns were smaller in Design D than those resulting from Design A. The beam sizes were identical to those sizes resulting from Design B. The total weight for the structure was approximately 381,800 pounds, the lightest of all example frames.

The results of a second order elastic-plastic analysis of Design D are shown as the solid line in Figure 7.7. The first hinge formed at $\lambda = 1.06$ with the ultimate conditions reached at $\lambda = 1.23$. The resulting structure had an overall working load sway index of 0.00247, approximately 9% greater than the sway index for Design A. At ultimate load, there were 46 beam hinges and 39 column hinges. The hinge pattern is shown in Figure 7.8. The frame underwent considerable

deformation before failure.

7.6 DESIGN OF A LEANED FRAME

To examine the behavior of a more extreme structural form designed using the PΔ technique, the example frame was re-designed. In this case, however, it was assumed the frame supported laterally the two adjacent frames. The rigid frame carries all lateral forces through the bending resistance of the beam to column joints. In the adjacent frames, the beam to column joints were designed to transfer only shear. Thus, the adjacent frames are only designed to carry vertical loads.

The lateral shears applied to the non-rigid frames, whether wind loads, or second order shears are transferred to the rigid frame by the diaphragm action of the floors. They accentuate the stability effects in this problem, the effective wind force was reduced by a factor of three. Thus, in spite of the fact that the central rigid frame is required to carry all lateral forces applied to the flexible frames, the total design lateral load applied to the frame was the same as the other examples.

The analysis of the frame is facilitated by linking the rigid frame to a flexible column that carries all of the vertical load applied to the flexible frames. The second order effects from the flexible frame are thus passed to the supporting frame by a link. This procedure was used by Davison for the analysis of leaned sub-assemblages (14). The model used for the analysis was later confirmed by modifying Davison's program to include the second order effects of

the leaned frame directly in the shear equilibrium equation. The results of the analyses using this modified program were essentially the same as those obtained from the leaned frame model.

Since the second order effects for the flexible frame have been accounted for, along with the applied wind forces in the design of the rigid frame, the flexible frame may be designed as a sway prevented frame. A design such as this would not be allowed using traditional procedures. The rigid frame was designed on the basis of a second order analysis of the linked frame model. A check of the sway effects ratio, R1, defined in Chapter V, showed this value varied between 0.40 and 0.42 for storeys of the frame. These values approach the limit suggested in Chapter V. The member sizes for the rigid frame are shown in Table 7.2. The resulting member sizes show a significant increase in column sizes.

The resulting frame was then analyzed using the second order elastic-plastic analysis for combined loading. The results of this analysis are shown in Figure 7.9. The first hinge developed at λ = 1.18, an acceptable value for limit states design. The structure failed suddenly due to overall instability at λ = 1.21. At ultimate load, there were nine girder hinges and five column hinges as shown in Figure 7.10.

At working loads, the first order sway index, Δh for the structure was 0.0168, with a maximum storey sway, Δ/h, of 0.00189. These values are within acceptable limits. The maximum second order sway for the structure at working loads, Δh, was 0.00028, slightly

higher than an acceptable lateral sway (24). It is possible that an unusual structure with significant sway effects may have an acceptable sway index. Thus, the suggested check of the stability ratio appears to be justified, although continued research into this type of failure is warranted.

7.7 SUMMARY

In this chapter, a number of different design techniques were applied to the same frame. A comparison of the resulting designs and their behavior as predicted by a second order elastic-plastic analysis showed the following:

a) Design A, a limit states design using the $P\Delta$ technique, results in the lighter structure when compared to traditional design procedures. This structure performs quite satisfactorily under both specified and factored loads. This design does result in a collapse condition that is predominantly column hinges, with less capacity to absorb energy.

b) Design B, a limit states design with traditional provision for sway effects, results in a somewhat heavier structure than Design A. Design B, however, is somewhat more flexible and forms hinges at a factor of safety less than required by the standard. The collapse condition, however, is predominantly beam hinges with more capacity to absorb energy.

c) Design C, a traditional allowable stress design, is considerably heavier than Design A. Design C does perform satisfactorily at specified and factored loads. In addition, this design has a somewhat higher capacity to absorb energy before collapse.

d) Design D, a design procedure in which second order effects are neglected if certain conditions are met, results in the lightest structure, if the axial load ratio is ignored. If this ratio is applied as suggested, the resulting column sizes were unrealistically larger than the other designs. It must be noted that this axial load ratio was arbitrarily assigned in the original study and a rational discussion of this ratio is not possible. The analysis of the resulting frame showed a somewhat more flexible structure that did not satisfy limit states criteria and reached ultimate conditions at a significantly lower load factor than other designs.

In addition to these comparitive designs, an example of a "leaned frame" design was performed. The leaned frame was designed to develop significant sway effects beyond the limits suggested in Chapter V. While the structure satisfied limit states criteria, the sudden stability failure was developed just past the limit state. The results of this example tends to confirm the concern over structures with stability problems.

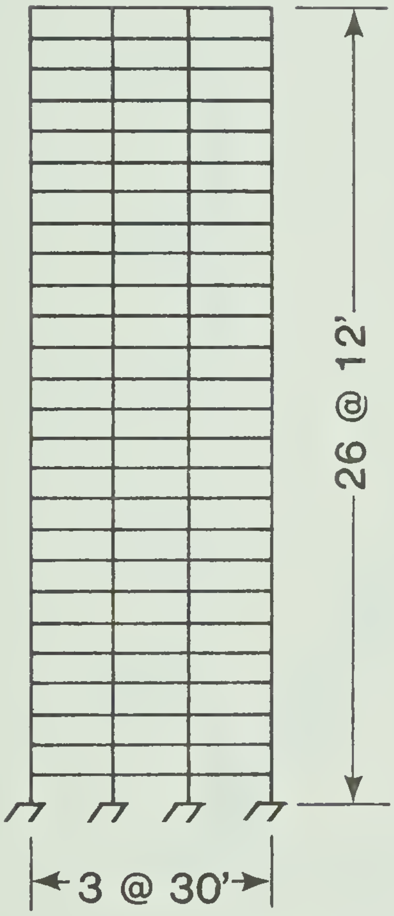
TABLE 7-1 MEMBER SIZES FOR DESIGN EXAMPLES

	Beams				Columns				
Levels	Design A	Design B	Design C	Design D	Levels	Design A	Design B	Design C	Design D
2	W27x84	W24x84	W27x84	W24x84	1-3 EXT	W14x342	W14x342	W14x342	W14x314
3	W27x84	W24x84	W27x84	W24x84	INT	W14x398	W14x426	W14x426	W14x298
4	W27x84	W24x84	W27x84	W24x84	3-5	W14x287	W14x314	W14x314	W14x264
5	W27x84	W24x84	W27x84	W24x84		W14x342	W14x370	W14x398	W14x342
6	W27x84	W24x84	W27x84	W24x84	5-7	W14x246	W14x287	W14x287	W14x237
7	W27x84	W24x84	W24x84	W24x84		W14x314	W14x342	W14x342	W14x314
8	W24x84	W24x84	W24x84	W24x84	7-9	W14x228	W14x246	W14x246	W14x211
9	W24x84	W24x76	W24x84	W24x76		W14x287	W14x314	W14x314	W14x287
10	W24x84	W24x76	W24x84	W24x76	9-11	W14x202	W14x219	W14x228	W14x193
11	W24x84	W24x76	W24x76	W24x76		W14x246	W14x287	W14x287	W14x264
12	W24x76	W24x76	W24x76	W24x76	11-13	W14x184	W14x202	W14x202	W14x167
13	W24x76	W24x68	W24x76	W24x68		W14x219	W14x246	W14x246	W14x246
14	W24x76	W24x68	W24x76	W24x68	13-15	W14x158	W14x176	W14x176	W14x150
15	W24x76	W24x68	W24x68	W24x68		W14x193	W14x211	W14x219	W14x193
16	W24x68	W24x68	W24x68	W24x68	15-17	W14x136	W14x150	W14x158	W14x127
17	W24x68	W24x68	W24x68	W24x68		W14x167	W14x184	W14x184	W14x167
18	W24x68	W24x61	W24x68	W24x61	17-19	W14x119	W14x127	W14x136	W14x111
19	W24x61	W24x61	W24x68	W24x61		W14x142	W14x150	W14x158	W14x142
20	W24x61	W24x61	W24x61	W24x61	19-21	W14x103	W14x103	W14x111	W14x103
21	W24x61	W24x55	W24x61	W24x55		W14x111	W14x119	W14x127	W14x111
22	W24x55	W24x55	W24x61	W24x55	21-23	W14x 78	W14x 84	W14x103	W14x 78
23	W24x55	W21x55	W24x61	W21x55		W14x 84	W14x103	W14x103	W14x103
24	W24x55	W21x55	W24x61	W21x55	23-25	W14x 61	W14x 61	W14x 68	W14x 61
25	W21x55	W21x55	W24x61	W21x55		W14x 61	W14x 61	W14x 68	W14x 61
26	W21x55	W21x55	W24x61	W21x55	25-27	W10x 49	W8x 48	W10x 49	W14x 48
27	W18x40	W21x44	W24x49	W21x44		W8x 35	W8x 40	W8x 40	W8x 35

TABLE 7.2

MEMBER SIZES FOR LEANED FRAME

Beams		Columns	
Levels	Sizes	Levels	Sizes
2	W30x99	1-3 EXT	W14x342
3	W30x99	INT	W14x398
4	W30x99	3-5	W14x314
5	W30x99		W14x342
6	W30x99	5-7	W14x287
7	W30x99		W14x342
8	W30x99	7-9	W14x246
9	W27x94		W14x314
10	W27x94	9-11	W14x219
11	W27x94		W14x287
12	W27x94	11-13	W14x193
13	W27x84		W14x246
14	W27x84	13-15	W14x176
15	W24x84		W14x219
16	W24x84	15-17	W14x150
17	W24x76		W14x184
18	W24x76	17-19	W14x127
19	W24x68		W14x150
20	W24x68	19-21	W14x103
21	W24x61		W14x127
22	W24x61	21-23	W14x 84
23	W21x55		W14x103
24	W21x55	23-25	W14x 61
25	W21x55		W14x 68
26	W21x55	25-27	W14x 48
27	W21x44		W14x 48



Story-Bay 26-3

Spacing 30 ft.

Roof Load
Live 30 psf
Dead 40 psf
Floor Load
Live 75 psf
Dead 50 psf
Wall (Story) 18 kips
Wind Load 20 psf

Figure 7.1 Frame Geometry And Loading

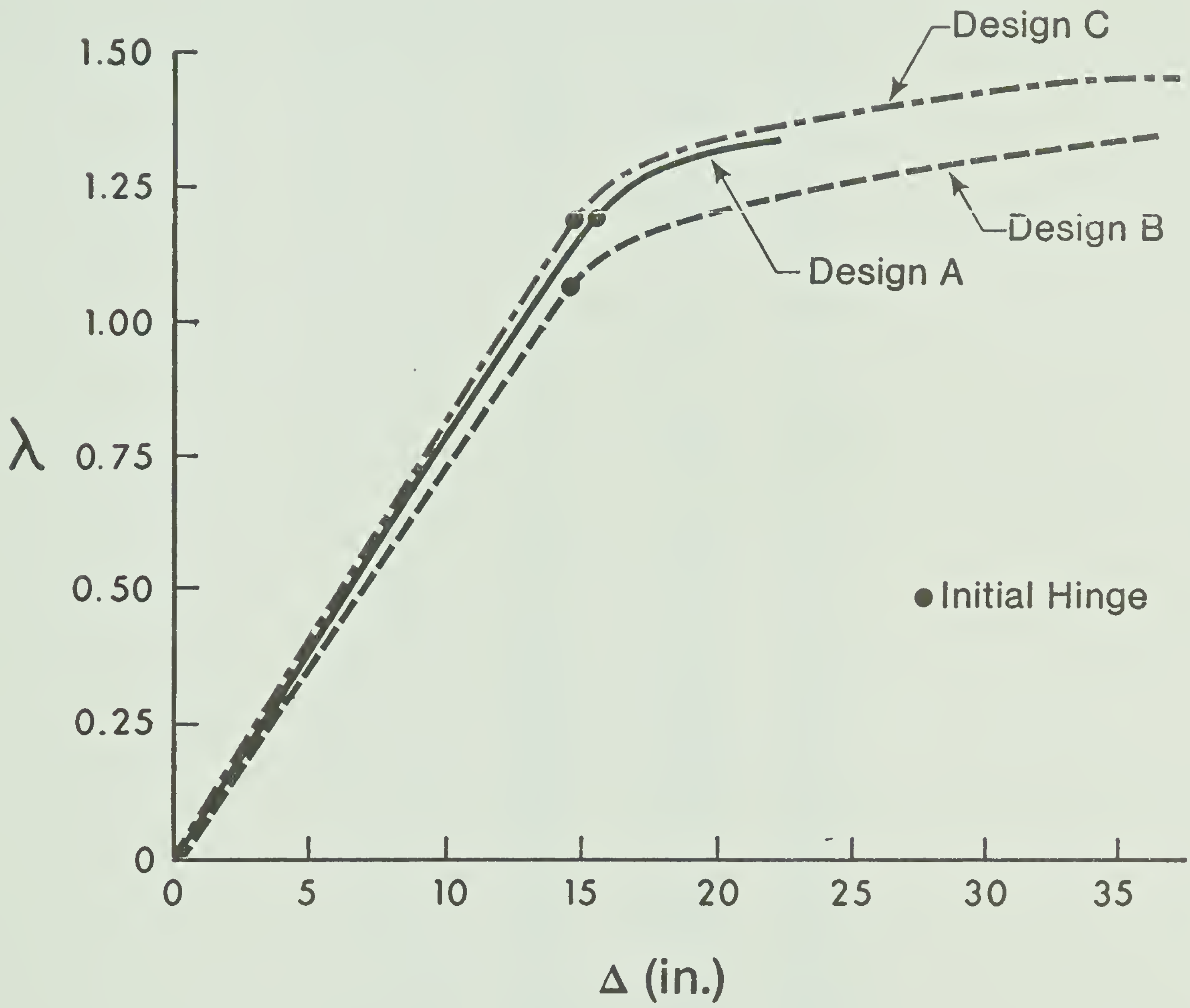


Figure 7.2 Load Factor-Lateral Deflection Relationships For Combined Loading

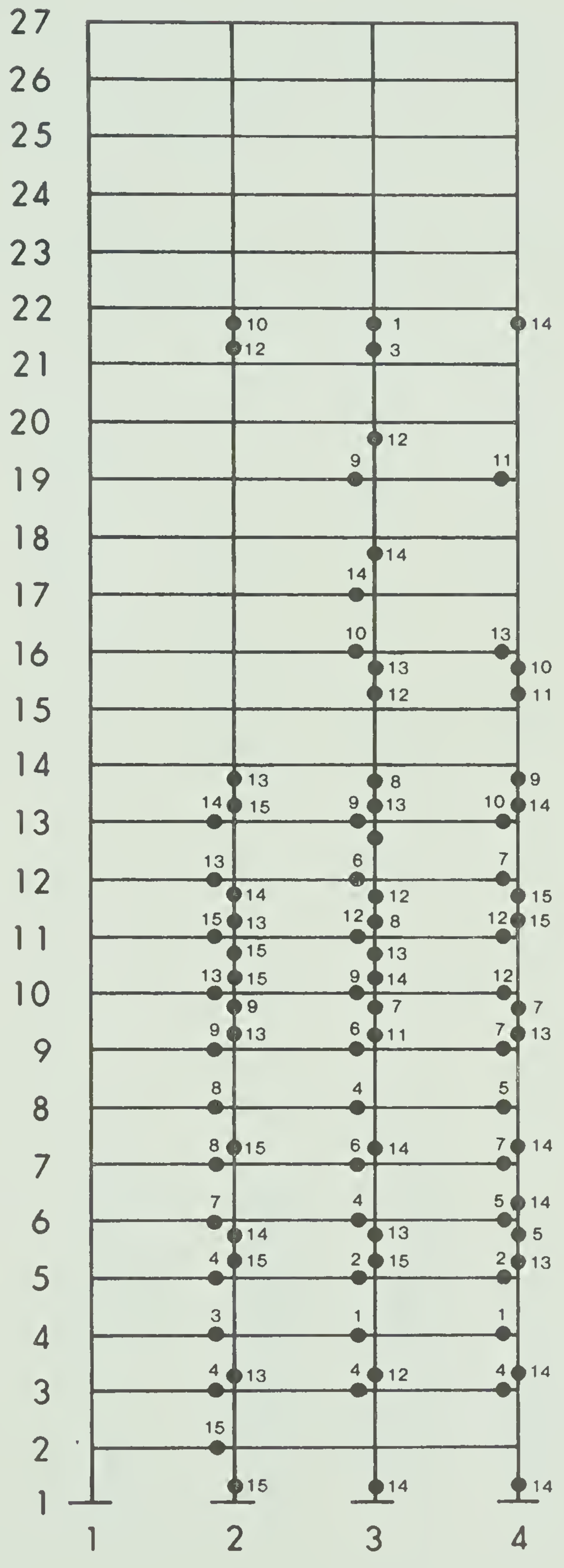


Figure 7.3 Hinge Pattern For Design A Combined Loading

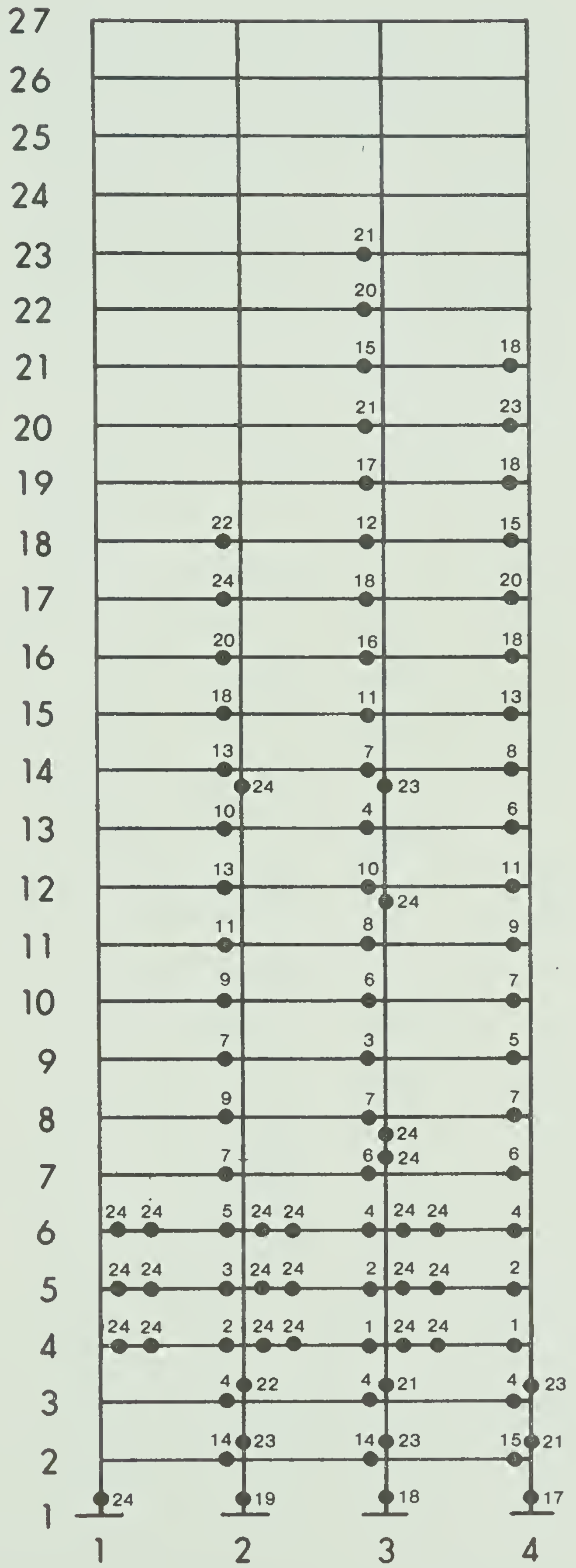


Figure 7.4 Hinge Pattern For Design B Combined Loading

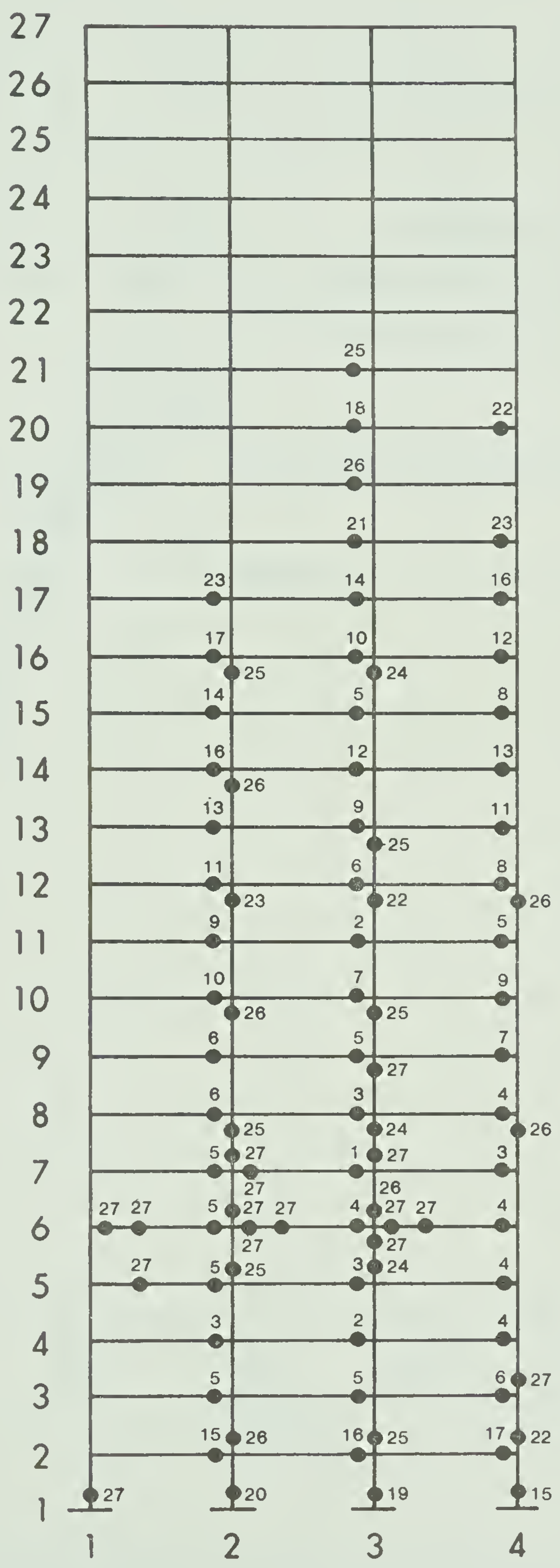


Figure 7.5 Hinge Pattern For Design C Combined Loading

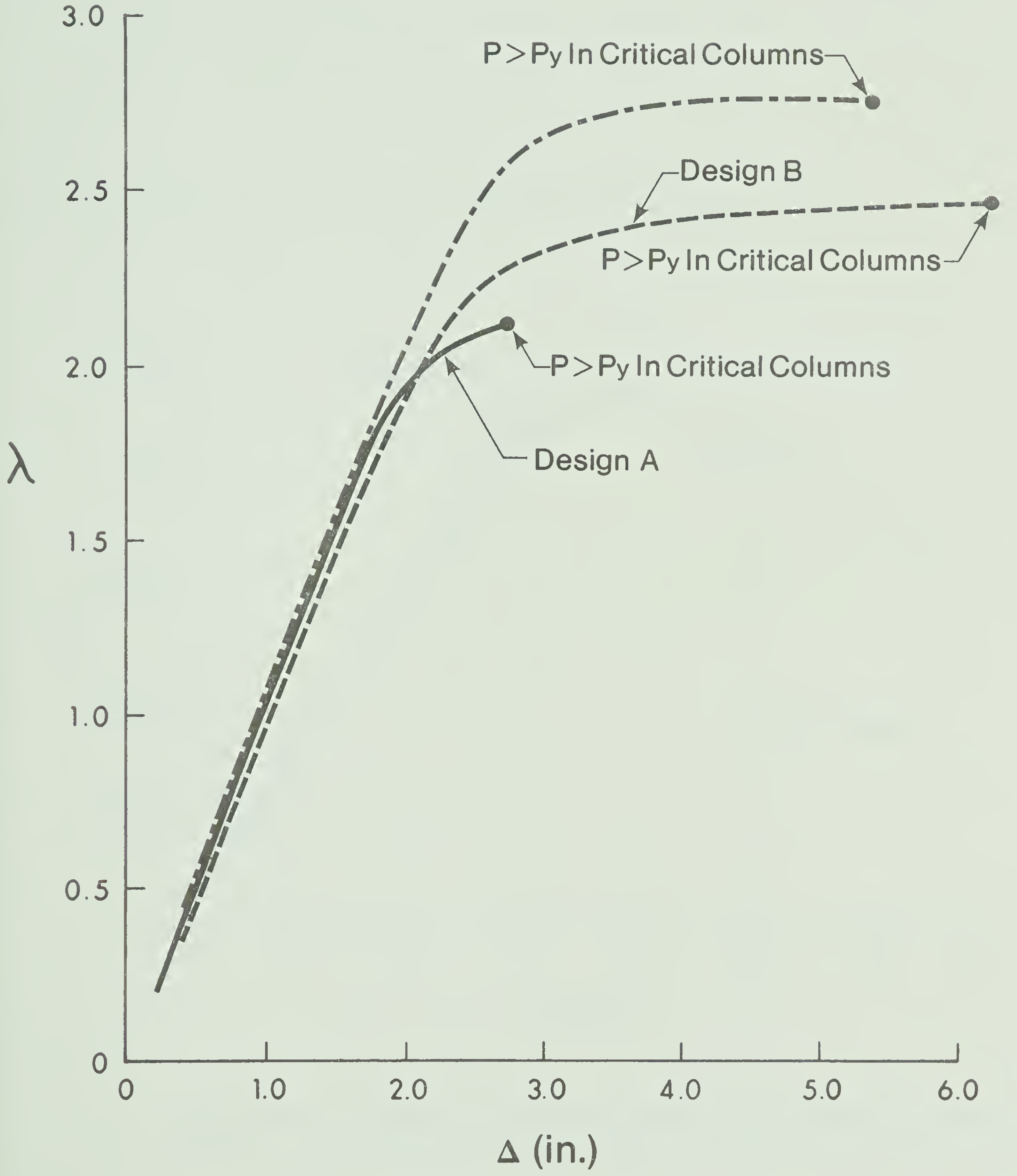


Figure 7.6 Load Factor-Lateral Deflections Relationships For Vertical Loads Only

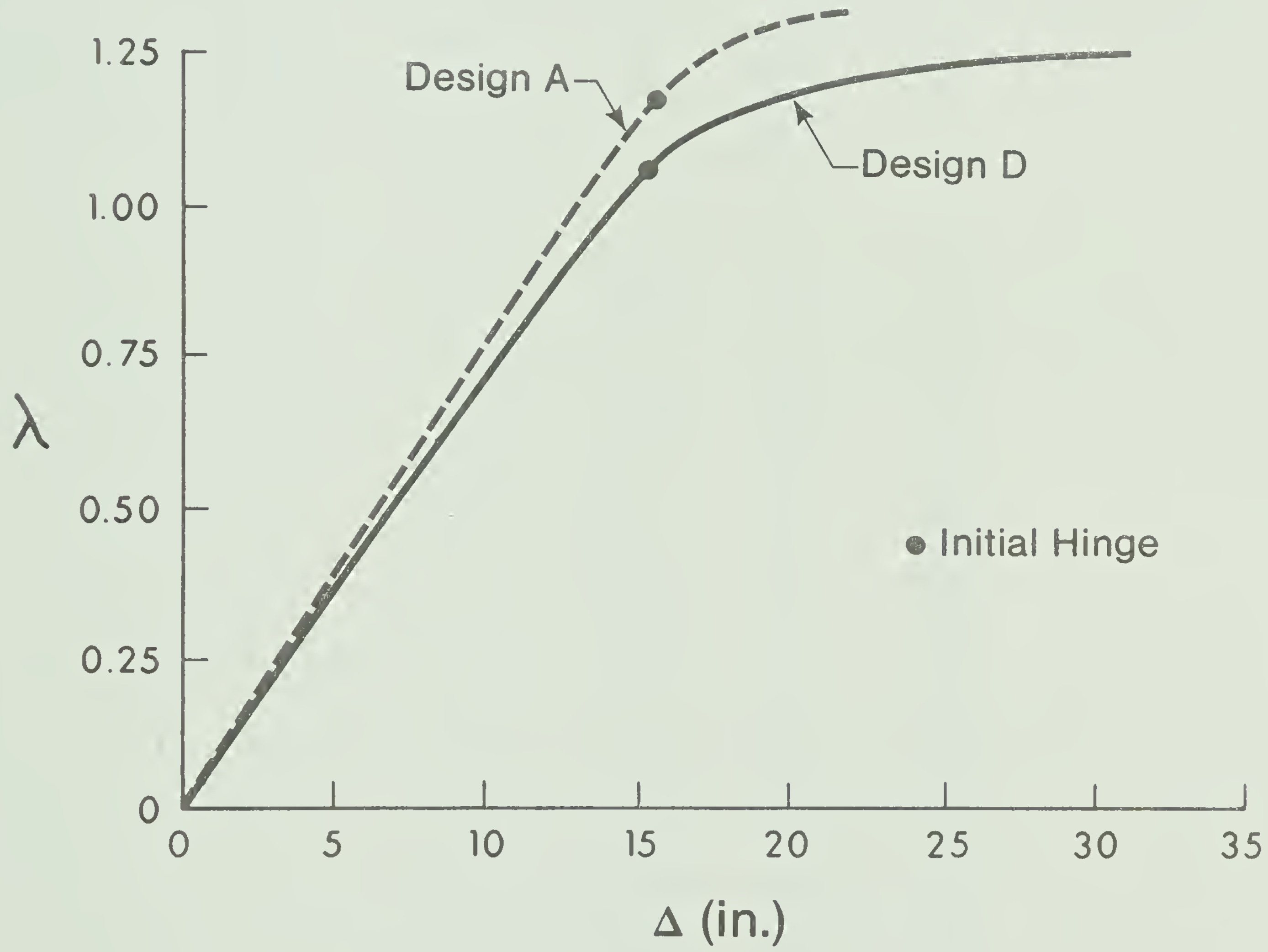


Figure 7.7 Load Factor - Deflection Curve For Design D, Combined Loading

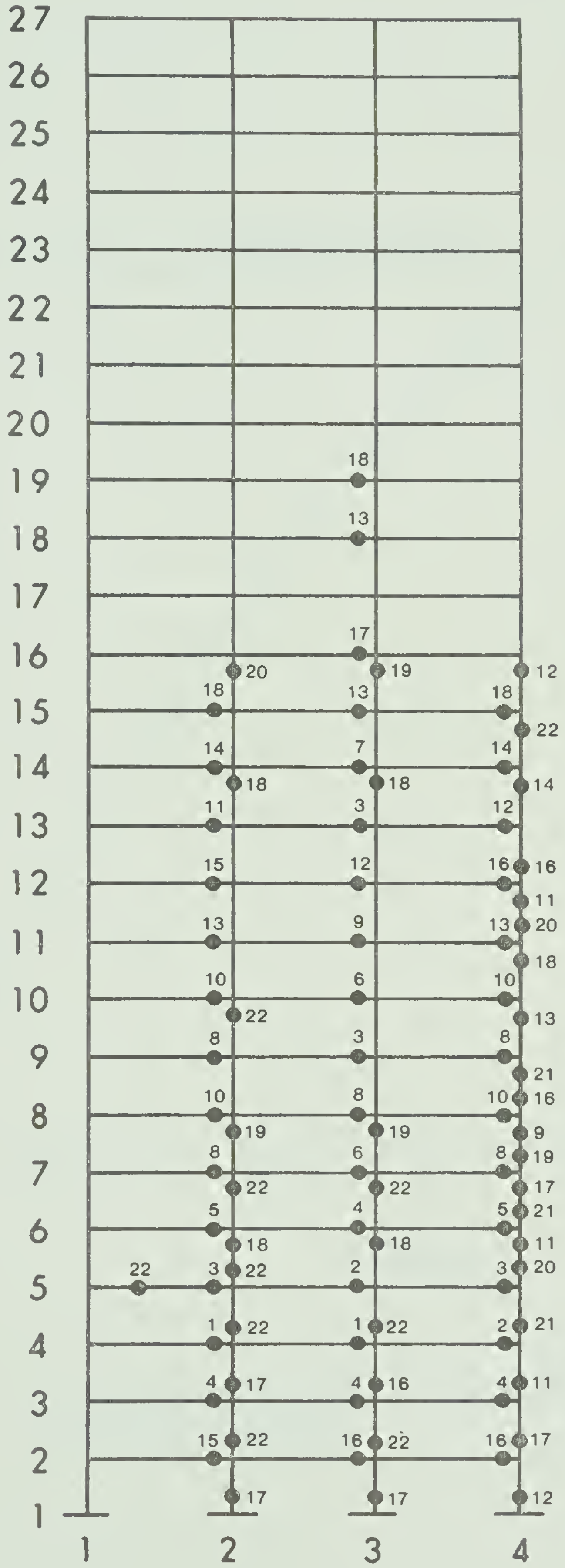


Figure 7.8 Hinge Pattern For Design D, Combined Loading

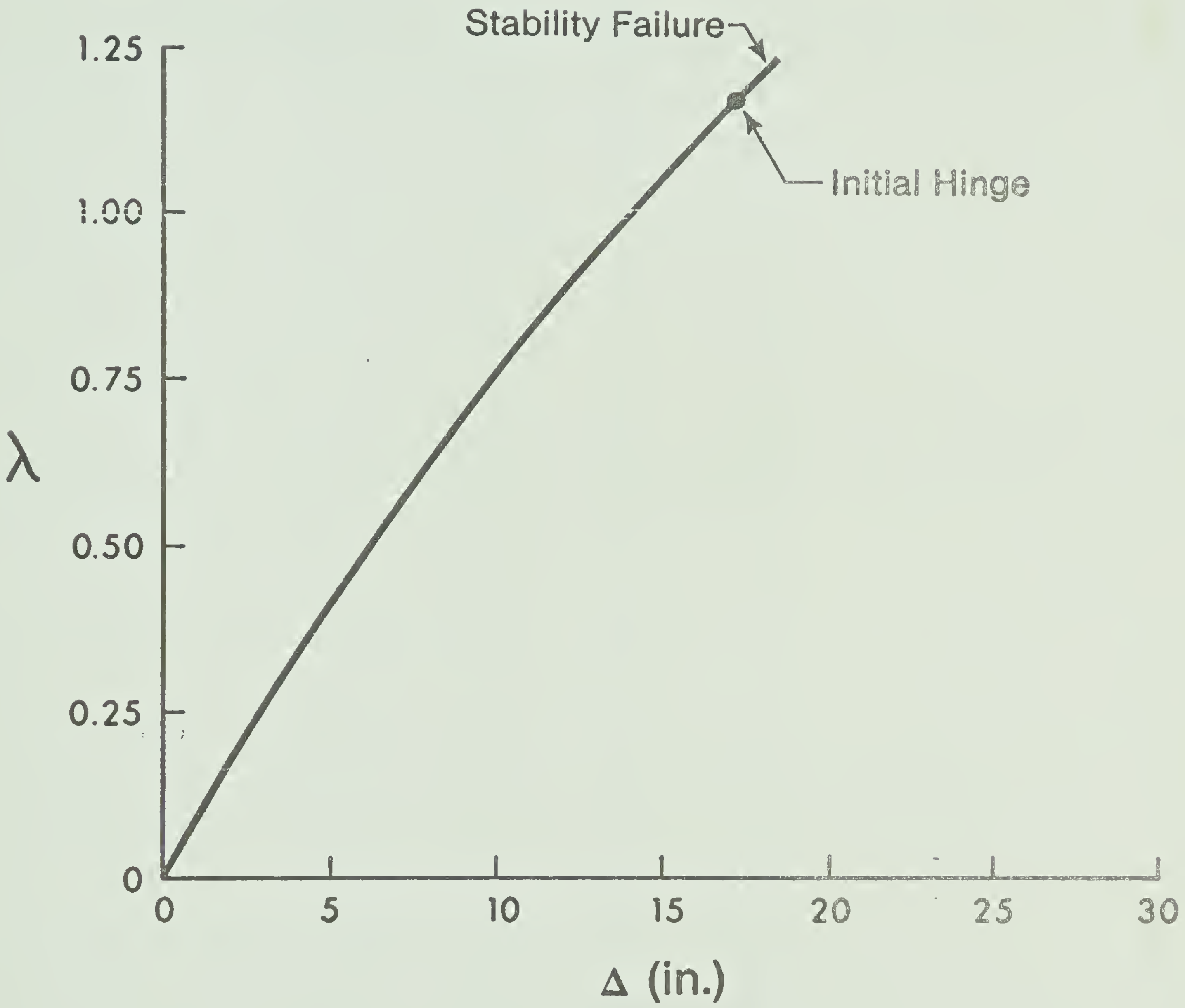


Figure 7.9 Load Factor - Deflection Relationship For Leaned Frame, Combined Loading

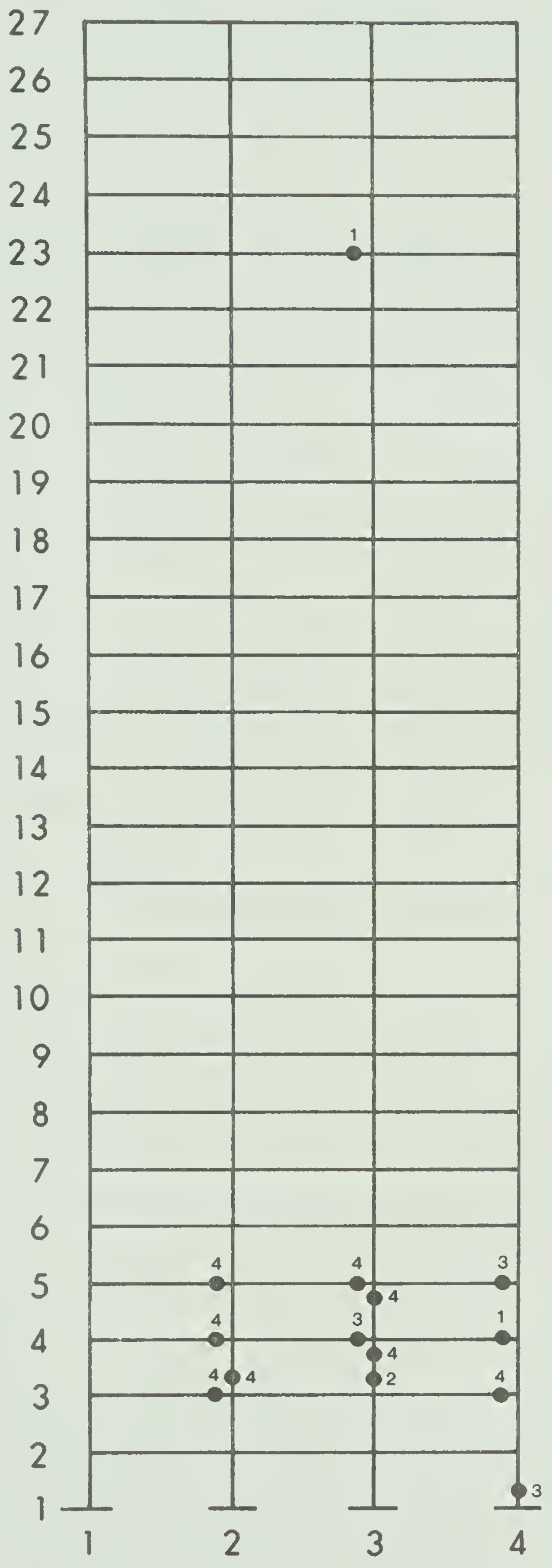


Figure 7.10 Hinge Pattern For Leaned Frame, Combined Loading

CHAPTER VIII

DISCUSSION AND RECOMMENDATIONS

8.1 INTRODUCTION

The problems related to the design for stability of steel structures have been outlined in the previous chapters along with a number of proposed design methods. Particular emphasis has been placed on the design of structures using the $P\Delta$ procedure and the use of the interaction equations (Equation 3.20 and 3.21) to account for the stability of structures.

The object of this chapter is to draw together the information presented in the previous chapters and develop recommendations for the design of steel structures for stability.

8.2 DESIGN OF STEEL STRUCTURES FOR STABILITY

As outlined in previous sections, the main problem when designing a structure is to relate the design of a particular member to the overall stability of the structure. Traditionally, sway effects have been neglected when performing the analysis of the structure. As outlined in Chapter II, the sway effects can be significant when predicting the behavior of structures whether those structures are braced or unbraced. When sway effect ($P\Delta$ effects) are included in the analysis, the resulting prediction of the behavior of frames subjected to both combined loading and vertical loads only is found to be extremely close to behavior predicted by experiments.

The discrepancy between the behavior of a real frame and the results of a first order analysis is traditionally accounted for by the distinction between a "sway prevented" and a "sway permitted" structure and the resulting K factor and ω factor. The K factor is also used to account for effect of end restraints on the behavior of a beam column. The K factor results from the use of a mathematical model to predict the buckling of an individual member of the structure, while considering the influence of other members in the structure. There is, however, no relationship between the mathematical model used to predict the sway permitted K factors and the second order moments developed in the frame. Thus, the use of the K factor in the interaction equation (Equation 3.21) amounts to an empirical prediction of the ultimate strength of a restrained column permitted to sway.

In effect, each individual member will resist the factored loads without failure due to overall instability or local material failure. This implies that the members in that storey will not fail as a unit due to an overall instability. There is, however, no specific check of the stability of the structure itself.

8.3 PROPOSED MODIFICATIONS TO TRADITIONAL PROCEDURES

As outlined in previous chapters, the model traditionally used to calculate the effective length factor, K, is based on a series of simplifying assumptions. Thus, K, when based on the sway permitted case, reflects only the effects of end conditions for an isolated member when the structure is on the verge of buckling in a sidesway mode. Thus, a number of proposals have been developed as attempts to reflect

in some way, the overall buckling of the frame itself.

Edmonds and Medland suggested that the effective length factors could be calculated on the basis of the buckling of the frame itself (6) as shown in Chapter III. A comparison between the interaction predicted by an ultimate strength analysis and the interaction predicted by Equation (3.21) with K based on the frame buckling condition showed the proposal to be unrealistically conservative. This discrepancy arises from the fact that the stability of a beam column is not related to a mathematical prediction of the critical load for a perfect frame. The frame buckling load is often related to the behavior of a specific storey in the frame, and thus, cannot be expected to relate to the behavior of members far removed from the area of weakness.

An apparently more rational approach would be to consider the buckling of each storey separately and calculate K based on the buckling load of each storey. A comparison between the interaction predicted when a K factor is based on a storey buckling and the ultimate strength of the member shows that, in some cases, the storey buckling K is over-conservative, and in other cases, this factor results in an unconservative prediction of member strength. Thus, the resulting structure may satisfy overall stability requirements, while not satisfying individual member strength. For Limit States Design, this procedure would be unacceptable (24).

Lay suggested a procedure that would consider the effect of axial load on member stiffness when considering the restraint provided to the ends of a column (5,30). At present, members with axial load above and below the column in question are assumed to be on the

verge of buckling and thus, share the restraint provided by the beams to the beam-column joint. While Lay's proposal will reduce the stiffness of columns framing into the joint, it will never result in a negative stiffness as presently assumed. When a K factor calculated by Lay's procedure is used in Equation (3.21), the interaction predicted is unconservative when compared to the results of an ultimate strength analysis. This is another case in which the adjustment of the K factor did not consider the actual use of this factor in practice.

Lay also proposed a method for classifying members in structures as sway prevented in structures without stiff vertical bracing elements (5,30). While studies in Chapter IV showed that when considerations frame buckling of one storey can result in members that are effectively sway prevented, in fact, other members in the same storey that are supporting the more flexible columns may weaken by the resulting leaning action. Before a member can be classified as sway prevented on the basis of support provided by other members, some provision of the effect this action has on the supporting members must be made to ensure overall stability of the frame. As outlined when dealing with Medland's suggestions, the requirements of a limit states design procedure may not be met when a re-distribution of load carrying capacity takes place in the frame (24).

Yura developed proposals for the adjustment of the effective length for inelastic column behavior and for the consideration of overall frame instability. As shown in Chapter IV, the use of a K factor based on inelastic column behavior results in a reasonable prediction of beam column strength for stockier columns often used in the design

of buildings of medium height. In other cases, however, the prediction remained quite conservative.

While Yura's initial proposal for moment magnification in mixed framing systems provided a reasonable prediction of the ultimate strength of the supporting member, the stability check as proposed in Equation 4.14 does not reflect the actual stability of the structure. The stability of a structure is not related simply to the sum of the axial load resistances of the members, but the ability of the structure to resist sway effects.

In each of these proposals, the approach has been to attempt to provide a more rational model for the calculation of the effective length factor. Each proposal would increase considerably the calculations required for the design of beam columns. Yet, the proposals do not provide a consistent allowance for the second order moments in beam columns or provide any allowance for second order moments in beams. There is no certainty that any of these proposals will actually ensure overall stability.

8.4 THE PΔ PROCEDURE

The alternative to a design based on the results of a first order analysis corrected by use of effective length factors would be direct calculation of second order moments as outlined in Chapter V. If the second order effects have been included in the analysis, than the members may be designed for member strength and stability as predicted when the interaction equations are used with the sway prevented K and ω factors.

A comparison between this prediction and the column end moments at ultimate strength of a restrained column permitted to sway shows that the member may not attain the predicted end moments before failure due to member instability. This discrepancy between predicted member strength and actual member strength has been shown to increase when the effect of inelastic beam response is considered in Chapter VI. The inelastic response of beams was found to only have a significant effect on restrained columns that are susceptible to a stability failure and only when beam yielding corresponds to the column ultimate strength.

The discrepancy between the ultimate strength predicted by the $P\Delta$ procedure and the ultimate strength predicted by analysis arises when the member fails due to instability before the ultimate moment of member is reached. This instability results when second order effects become significantly large compared to member strength and stiffness. Analyses of restrained columns in Chapter V have shown that this instability can be predicted by comparing sway effects to applied loads.

When an approximate second order analysis is used, the possibility that a stability problem exists may be examined by calculating the ratio of increase in shears due to second order effects to the applied lateral shears. If this ratio is greater than 0.50 for any storey, that storey can be defined as unstable and member sizes must be increased to provide more stability. When a direct calculation of second order effects is used (14,41), a first order analysis should also be performed and the same ratio should be calculated and checked.

Thus, when the $P\Delta$ procedure is used for design of members in a structure, the members will be designed for actual conditions that

exist at the limit state. The prescribed checks of sway effects at limit state will serve as a check of overall stability of the structure. The PΔ procedure provides a method of designing for member strength and checking for structural stability at the same time. It must be emphasized that the second order analysis must include any effect that can significantly effect deflections, and therefore, sway effects.

The design examples for a 26-storey building frame show that the PΔ procedure results in a structure that is lighter yet stiffer than the structure designed using the traditional effective length procedures. In fact, the frame designed using the traditional procedures reached a limit state before factored design loads were reached. Thus, the traditional procedure may result in unsafe designs unless provision is made to ensure moment re-distribution can take place. This problem may have been avoided in the past by the effect of higher apparent factors of safety that result when the allowable stress design technique is used, as in design example C. It must be noted, however, that the failure of the frame designed using the PΔ technique was related to a series of column hinges. Thus, the frame did not display the ability to absorb large amounts of energy before failure.

The design and analysis of a structure that developed significant sway effects showed that a problem with stability effects can exist for a "leaned structure" when the PΔ design technique is used. This example was not conclusive, however, possibly because of the difficulty in modelling the "leaned structure" for the analysis. Further study or experiments may be required in this area.

A proposal to neglect the PΔ effects altogether under certain

conditions was also studied. This study was quite difficult as the proposal dealt with allowable stress design and some aspects of the proposal were not well defined. If all limits are applied as suggested, there would be few structures that could be designed using this proposal except for a few low structures. A design example of a medium height structure using this proposal resulted in an unsatisfactory structure when one of the proposed limits was ignored. If, however, this limit had been applied, the resulting column sizes were unrealistically over-sized. This particular limit, the ratio of axial load to axial resistance for columns appeared to be quite arbitrarily assigned.

8.5 RECOMMENDATIONS

8.5.1 DESIGN RECOMMENDATIONS

On the basis of the results of this investigation, the following recommendations are made with reference to the Canadian Standard S16.1 "Steel Structures for Buildings - Limit States Design" (24).

Structures designed in accordance with Clause 8.6.1 and 8.6.2, for which sway effects produced by vertical loads acting on a structure in its displaced configuration, should only use effective length factors and equivalent moment factors based on the sway prevented condition if a stability check of the structure is satisfied. This stability check shall be satisfied for a specific storey if the ratio of the sum of artifical lateral loads produced at or above the floor level caused by the vertical forces acting through the first order displacement, divided by the sum of the applied lateral shears at or above the floor

level is less than 0.50. In equation form, this requirement is:

$$\frac{\Sigma H'_i}{\Sigma H_i} < 0.50 \tag{8.1}$$

where:

H'_i is the artifical lateral load produced at the ith storey above the floor in question due to the vertical loads acting through the first order displaced configuration of the frame.

H_i is the applied lateral force at the ith storey above the floor in question.

The artificial lateral load can be calculated by referring to Figure 8.1.

$$H'_i = V'_{i-1} - V'_i \tag{8.2}$$

where:

V'_i = the artificial shear in storey i due to sway forces is given by:

$$V'_i = \frac{\Sigma Pi}{hi}(\Delta i + 1 - \Delta i) \tag{8.3}$$

where:

ΣPi = sum of the column axial loads in storey i

h = height of storey i

$\Delta i + 1$; Δi = first order displacements of level i + 1 and i, respectively.

For the case of vertical loads only, the applied lateral forces shall be considered as the artificial forces caused by gravity loads acting on initially out-of-plumb members (47).

The second paragraph in Clause 8.6.1 of CSA S16.1 "Steel

Structures for Buildings - Limit States Design" (24), refers to certain structures for which sway effects may be neglected. It is recommended that this clause only be applied if the ratio

$$\frac{\Sigma H'_i}{\Sigma H_i} < 0.03 \qquad (8.4)$$

is satisfied, thus, the error in neglecting second order effects would be less than 3%.

It is also recommended that consideration be given to any phenomena that may have a significant effect on the deflections calculated in the second order analysis. Specifically, if the axial loads in the members are so large that

$$\sqrt{\frac{C_f L^2}{E\ I}} > 1.20 \qquad (8.5)$$

where:

C_f = the factored axial load in the member

L = the member length

E = Young's modulus

I = moment of inertia in the plane of bending for the member

then the effects of axial load on member stiffness should be considered.

In the cases when structures are designed using the provisions of Section 8.6.3 of CSA S16.1 "Steel Structures for Buildings - Limit States Design" (24), in which a first order analysis is used and sway effects are compensated for by use of the sway permitted K and ω factors, provision for beam hinging must be made. It is recommended that provisions be made that for structures designed in accordance with Clause

8.6.3 shall have beams that satisfy (24):

a) the width-thickness ratio requirements of a Class 1 section as given in Clause 11.2.

b) the steel used has $Fy < 0.80$ Fu and exhibits the load strain characteristics necessary to achieve moment distribution.

c) are laterally braced near the columns in accordance with Clause 13.7.

These provisions would assure that early beam hinges caused when second order effects in beams are neglected will not result in local failure before the specified load factor.

It is further recommended that in view of the material presented in Chapter IV, that when the results of a first order analysis is used for design, the traditional model for calculation of a sway permitted K factor be used. While this model provides results that are somewhat conservative and the model itself may not reflect actual structural behavior, the results provided by this model are apparently provide a relatively consistent factor of safety for individual member strength and structural stability.

8.5.2 RECOMMENDATIONS FOR FURTHER RESEARCH

With respect to the design of structures using the PΔ method, it is recommended that further design examples be studied to confirm the stability problem and check the value of the stability effects ratio suggested. To this end, it is suggested that large scale experiments of "leaned structures" be performed to further confirm the stabi-

lity problem. It is also suggested, that some of these tests be designed to produce significant stability effects due to inelastic action in restraining beams.

With respect to proposed modifications of the method for calculating the effective length factor, K, it is suggested that any modified K factor should be checked for its use in Equation (3.21). The interaction predicted by Equation (3.21) using the modified K should provide a reasonable agreement with the interaction predicted by an ultimate strength analysis of a restrained column permitted to sway as outlined in Chapter III. If the prediction provided is unconservative, evidence should be provided that the premature loss of lateral load capacity of one or more members of the frame does not affect the overall stability of the structure.

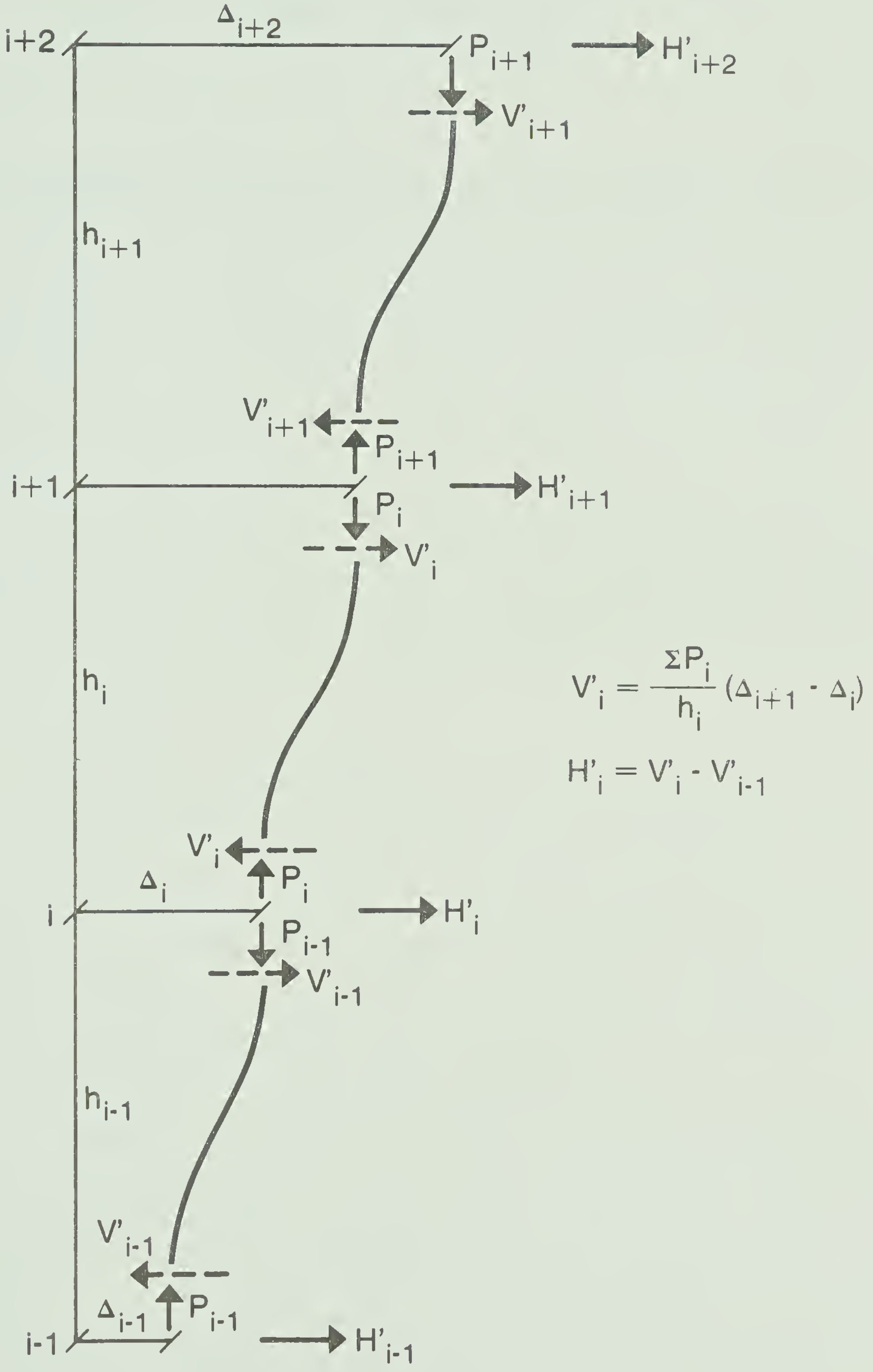


Figure 8.1 Sway Forces Due To Vertical Loads

CHAPTER IX

SUMMARY AND CONCLUSIONS

9.1 SUMMARY

Methods for the design of tall steel building frames for stability have been investigated in this thesis. First, the behavior of frames, both braced and unbraced, subjected to combined loading and vertical loads only was outlined. Various analytical models that can be used to predict the behavior of building frames were discussed.

It was shown that a first order analysis over-estimates the strength and stiffness of a structure, while a second order analysis (an analysis that includes $P\Delta$ forces) provides a reasonable prediction of structural behavior.

When a first order analysis is used, there must be an adjustment in the design procedure to account for the neglected second order effects. The traditional method for adjustment for second order effects has been presented, along with a number of proposed modifications to the traditional method. Analyses were performed to ascertain the effect of these modifications on the design of members.

A proposal to include second order effects in the analyses of structures, and thus, negate the adjustments required for design has been outlined. Analyses were performed to check the accuracy of the $P\Delta$ procedure. It was found that the $P\Delta$ procedure can be unconservative if the structure is approaching a stability failure. Limits were proposed to ensure the stability of structures designed by the $P\Delta$ procedure.

An analysis was developed to study the effect of inelastic beam response on the behavior of restrained columns permitted to sway. The inelastic beam response was found to reduce the apparent strength of restrained columns that are subject to stability failures. For members that fall within the limits proposed to measure sway effects, the effect of an inelastic beam response was found to be negligible.

An example building frame was designed using a number of different design procedures. The resulting members sizes were compared and the behavior of the resulting frames were predicted using a second order elastic-plastic analysis.

The results of the investigation were discussed and a number of recommendations were made for the design of steel structures for stability. Suggestions for the course of further studies were also put forth.

9.2 CONCLUSIONS

The PΔ procedure for the design of building frames provides for a rational design of steel frames as long as provision is made to assure the overall stability of the structure. Such a provision can be made by comparing the artificial shear forces predicted using the results of a first order analysis of the structure to the applied lateral forces. Although a structure that satisfies the criteria for lateral sway will not likely be subject to a stability problem, only a check of sway effects as outlined will ensure that stability criterion are satisfied.

The softening of member response caused by inelastic behavior

of beams was found to be only significant when the sway effects in the member are significant. Thus, when the structure satisfies criteria for structural stability, the limit state of the structure will not be influenced by inelastic action in beams caused by gradual penetration of the yielded zones.

Second order effects should only be neglected in the analysis if such effects are shown to be negligible in the analysis or if provisions are made in the design procedures to allow for neglected second order effects. An acceptable procedure for allowing for neglected second order effects is the use of sway permitted models for the calculation of effective length factors, K, and equivalent moment factors, ω. When this procedure is used, however, provision should be made for premature beam hinges since no allowance is made for second order moments in beams.

Since the use of a sway permitted effective length factor reflects an essentially empirical adjustment for neglected second order effects, the development of a more rational effective length factor does not necessarily result in a better prediction of member behavior. Proposed modifications to the effective length factor studied did not provide a more consistent prediction of member strength than the traditional effective length factor, when used in the stability interaction equation. Any future modifications of the traditional procedure for the calculation of effective length factors should be justified not be the rational of the effective length calculation, but by the use of this factor in design equations.

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APPENDIX A

CALCULATION OF STABILITY EFFECTS FOR ALLOWABLE STRESS DESIGN

An approximate analysis for the calculation of stability effects has been presented in Section 5.2. This analysis has been presented for the Limit States Design Technique. The analysis must be modified when using Allowable Stress Design. Equation A-1 is modified by the inclusion of a load factor, α, to:

$$V_i' = \frac{\alpha \Sigma P_i}{H_i} (\Delta_{i+1} - \Delta_i) \qquad \text{(A-1)}$$

where:

V_i = additional shear in story i due to the sway forces,

ΣP_i = sum of the column axial loads in story i,

α = the load factor,

H_i = height of story i, and

Δ_{i+1}, Δ_i = displacements of level i + 1 and i, respectively.

The sway forces due to the vertical loads, H_i, are then computed as the difference between the additional story shears at each level:

$$H_i' = V_{i-1}' - V_i' \qquad \text{(A-2)}$$

Equations (A-1) and (A-2) require that the sway forces be computed on the basis of the factored axial loads even though the purpose of the computation is to assess the magnitude of the PΔ effect at working loads.

This apparent inconsistency may be explained with reference to Figure (A-1), a plot of the load deflection relationship for a typical structure subjected to vertical loads, W, and horizontal loads, H.

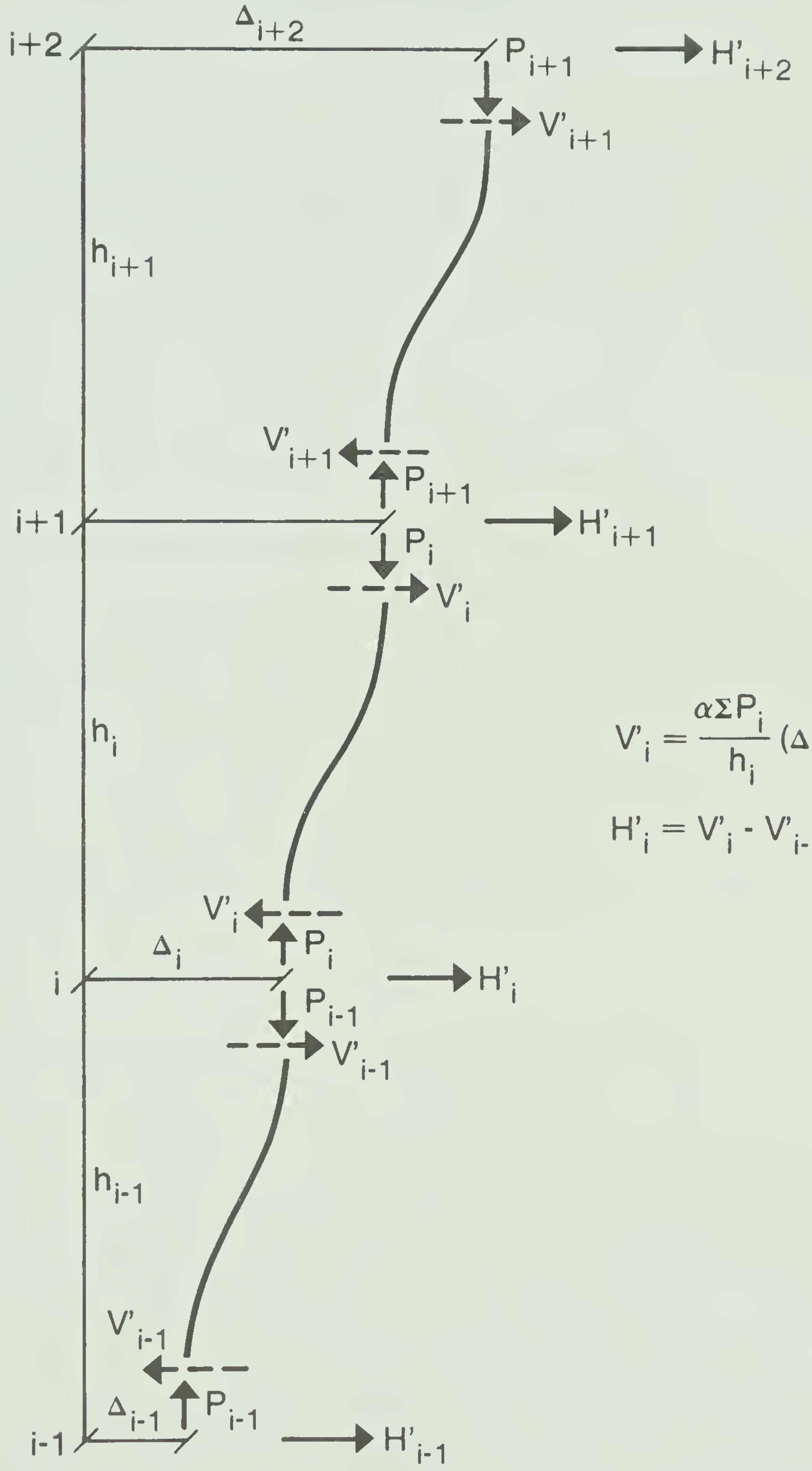
The response is assumed to be shown by the solid curve. Two stages on the curve are of interest; that at which the critical member reaches its ultimate capacity, corresponding to the lateral load, H_2; and the working load level, H_{2W}, respectively.

In the allowable stress design technique the ultimate capacity of the member is divided by a factor of safety, α, to arrive at the allowable capacity at the design load level. In North American Specifications this load factor is usually 1.70 and the implication in this technique is that since the resistance of the member can increase by a factor of $\alpha = 1.70$ between the working load level and the attainment of the ultimate capacity, the forces on the structure should increase by this same factor.

Since both the vertical and horizontal loads increase on the structure, the deflection corresponding to H_2, that is Δ_2, is more than 1.7 times the deflection Δ_{2W}. This is because, although the shears dues to the applied horizontal forces have increased by a factor of 1.7, the $P\Delta$ shears have increased by more than this amount since (see Equation A-1) both the vertical loads and the deflections increase during the loading history. However, if the story shears (Equation A-2) are computed on the basis of factored vertical loads, as suggested, this is equivalent to using

an increased deflection at working load level, Δ'_{2W} in Figure (A-1) in the computation of the PΔ forces. Under this procedure the story shears (and therefore, the bending moments and axial forces) corresponding to the attainment of the ultimate capacity in the critical member will be 1.70 times those at the working load level.

When using limit states design, however, there is no need to apply the load factor in Equation (A-1), as the load factor has already been applied in the analysis.



$$V'_i = \frac{\alpha \Sigma P_i}{h_i} (\Delta_{i+1} - \Delta_i)$$

$$H'_i = V'_i - V'_{i-1}$$

Figure A.1 Sway Forces Due To Vertical Loads

APPENDIX B

COMPUTER PROGRAM

The computer program presented herein for the inelastic analysis of restrained columns permitted to sway, was developed for an IBM 360 - model 67 computer with Michigan Terminal System. The program was developed for an interactive type of terminal use and is most efficient when used in this manner.

The interactive use allows a changing of analysis constraints to speed the analysis process. When the program was run on batch on an IBM 360 - model 50 computer at Lakehead University, the selection of the initial values for joint rotation and the joint rotation increments proved time consuming. The author and the University of Alberta disclaim any responsibility for the misuse of the program, nor will they be responsible for errors in the listing.

B.1 NOMENCLATURE FOR COMPUTER PROGRAM

ABM, ACM	- final girder end moments (K-in.)
ADM, ADMI	- column end moment, non-dimensionalized
ADMY	- column end moment (K-in.)
ALPHAX, ALX, ALX1	- vertical displacement at girder end divided by girder lengths
AXM	- girder end moment (K-in.)
AXV	- girder end shear (K)
BC	- curvature for moment-curvature relationship
BI, CI	- girder moment in inertia ($in.^4$)
BL, CL	- girder length (in.)
BM	- moment for moment-curvature relationship
BMB, BMC, BMF, BMX	- near end girder moment, non-dimensionalized
BMD	- bending moment at centre of girder segment, non-dimensionalized
BML, BMR	- far end girder moments, non-dimensionalized
BQ, CQ	- girder load (K/ft.)
BRM	- total girder resisting moment, non-dimensionalized
BS, CS	- girder plastic modulus ($in.^3$)
CM	- moment from column moment-end rotation relationship

CR	- end rotation from column moment-end rotation relationship
DA	- column area (in.^{2})
DELTA	- length of beam segment (in.)
DI	- column moment of inertia (in.^{4})
DIFF, DIFF2	- factors used to check convergence
DL	- column length (in.)
DM, DM2	- changes in girder end moments to force convergence
DP, P	- column axial load (K)
DTHA	- increment in girder-column joint rotation
BTHB, DTHC	- constant function used to relate far end to near end joint rotations
DTH1, DTH2, DTH3	- functions used to check for convergence
DZ	- column plastic modulus (in.^{3})
E	- Young's modulus (29000 KSI)
FMB, FMC	- trial girder end moments (K-in.)
FY	- yield strength (KSI)
GA	- column end rotation (radians)
H	- lateral shear resistance of subassemblage, non-dimensionalized
INDEX	- control index for far ends pinned case
NP	- number of points specified for analysis
NPB	- number of points in girder moment-curvature relationship

NPC	- number of points in column moment-end rotation relationship
PCM	- maximum column moment before column failure, non-dimensionalized
PHI	- curvature of girder segment
PHIYB, PHIYC	- girder yield curvatures, (radians)
PM	- column reduced plastic moment (K-in.)
PY	- column yield load (K)
PYB, PYC	- girder plastic moments (K-in.)
RM	- moment of horizontal resisting shear about far joint (K-in.)
RO	- joint sway (Δ/h)
RO1MP	- initial imperfection (Δ/h)
SIGN	- control sign for direction of integration
SIGN B, SIGN C	- control sign for convergence check
SLOPE	- slope of moment-curvature or moment-end rotation relationships
TH	- slope of girder segment
THA	- rotation of beam-column joint (radians)
THB, THC, THX	- rotation of joint at far end of the girder (radians)
TOL, TOL2	- tolerances for convergence checks
TX	- function used in convergence checks
XAM	- girder end moment (K-in.)
V	- deflection of girder segment (in.)

B2 Flow Diagram

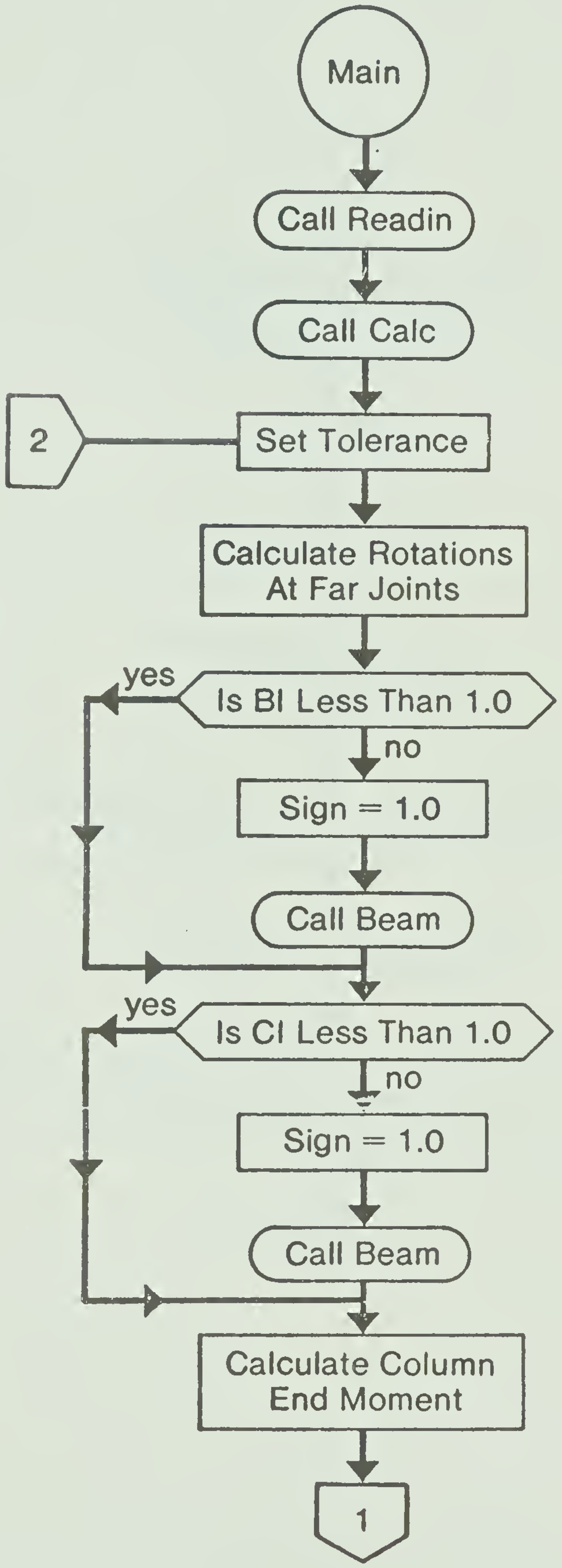


Figure B.1 Flow Diagram Main Program

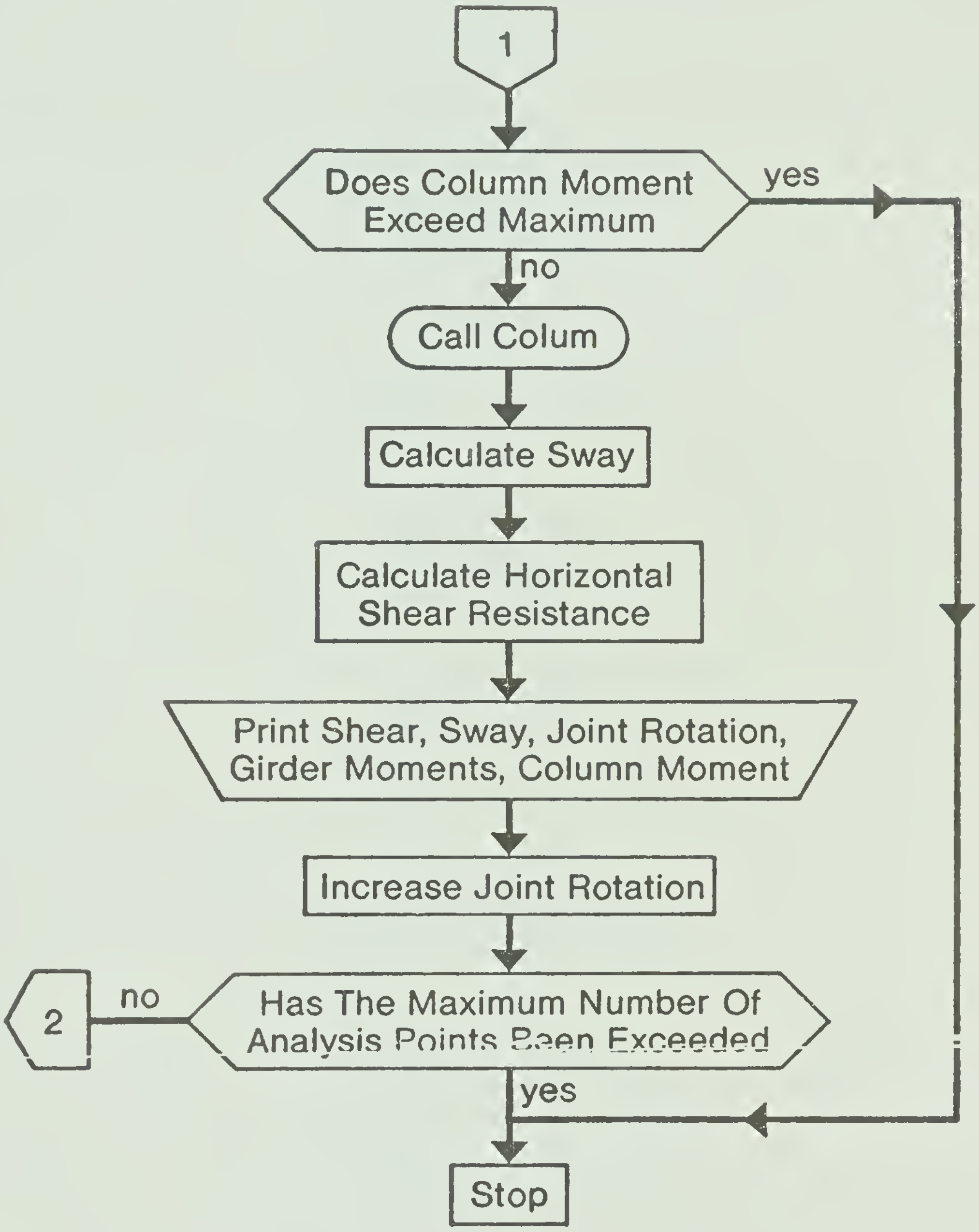


Figure B.1 Flow Diagram Main Program (continued)

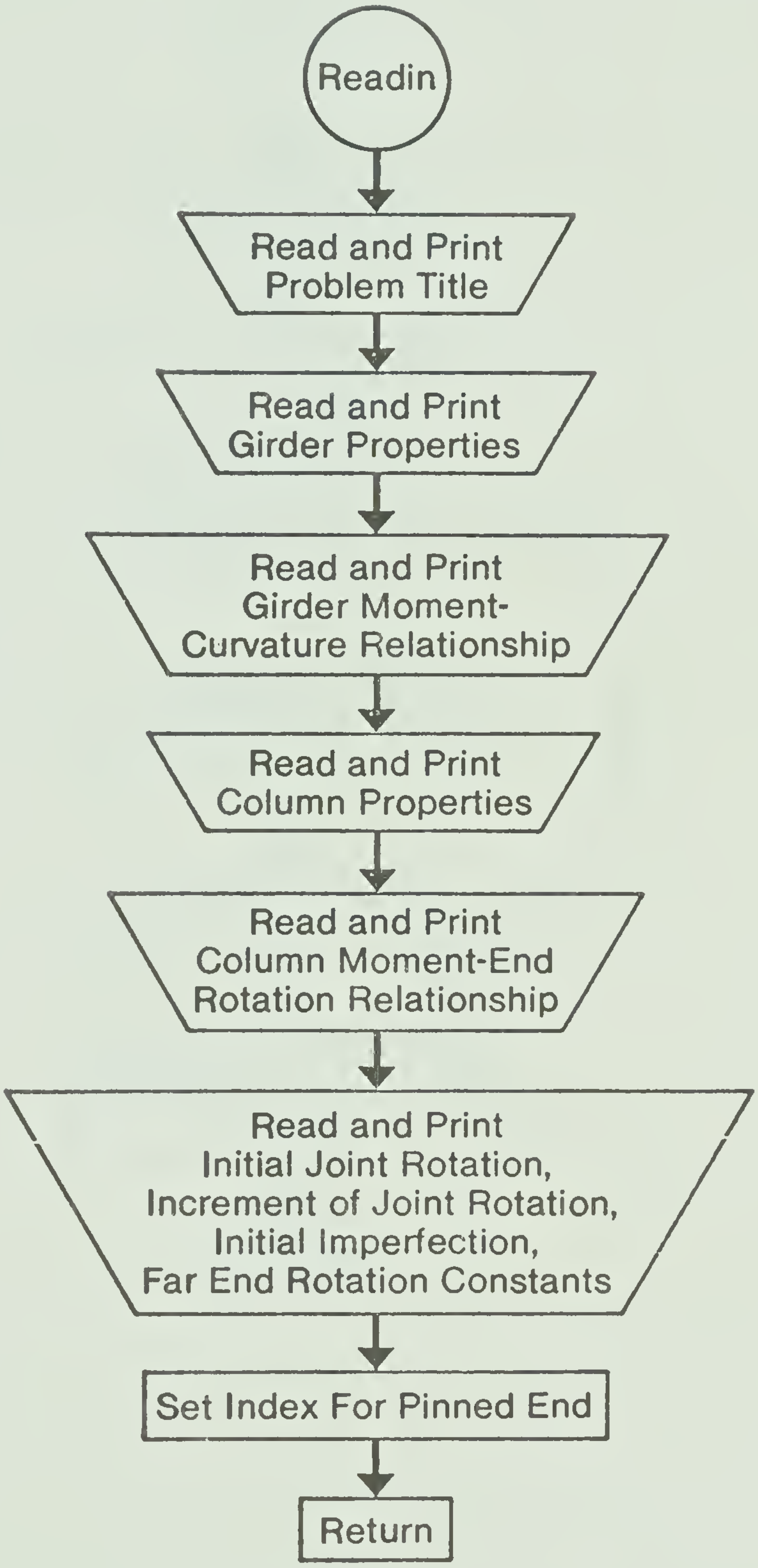


Figure B.2 Flow Diagram Subroutine Readin

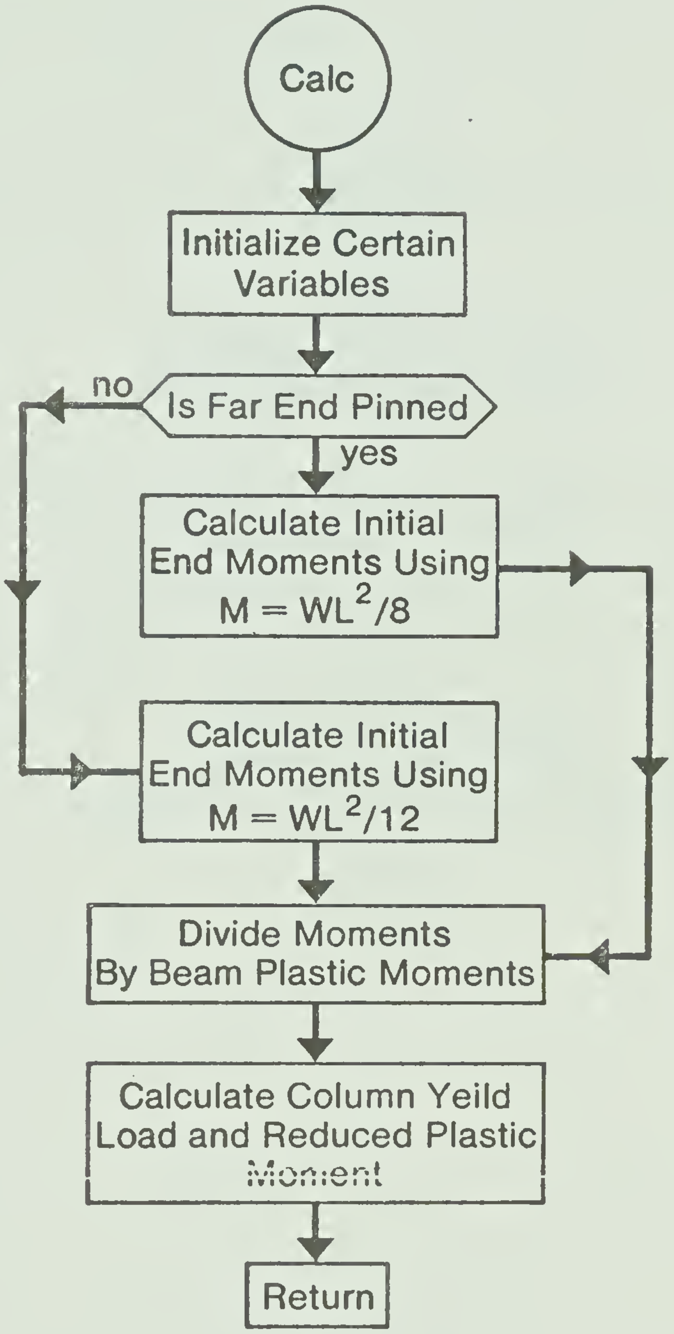


Figure B.3 Flow Diagram Subroutine Calc

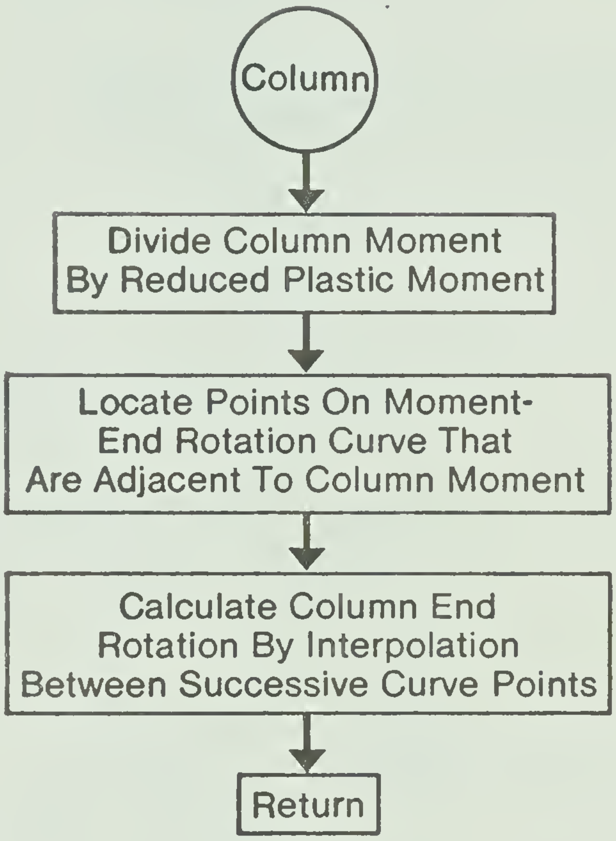


Figure B.4 Flow Diagram Subroutine Column

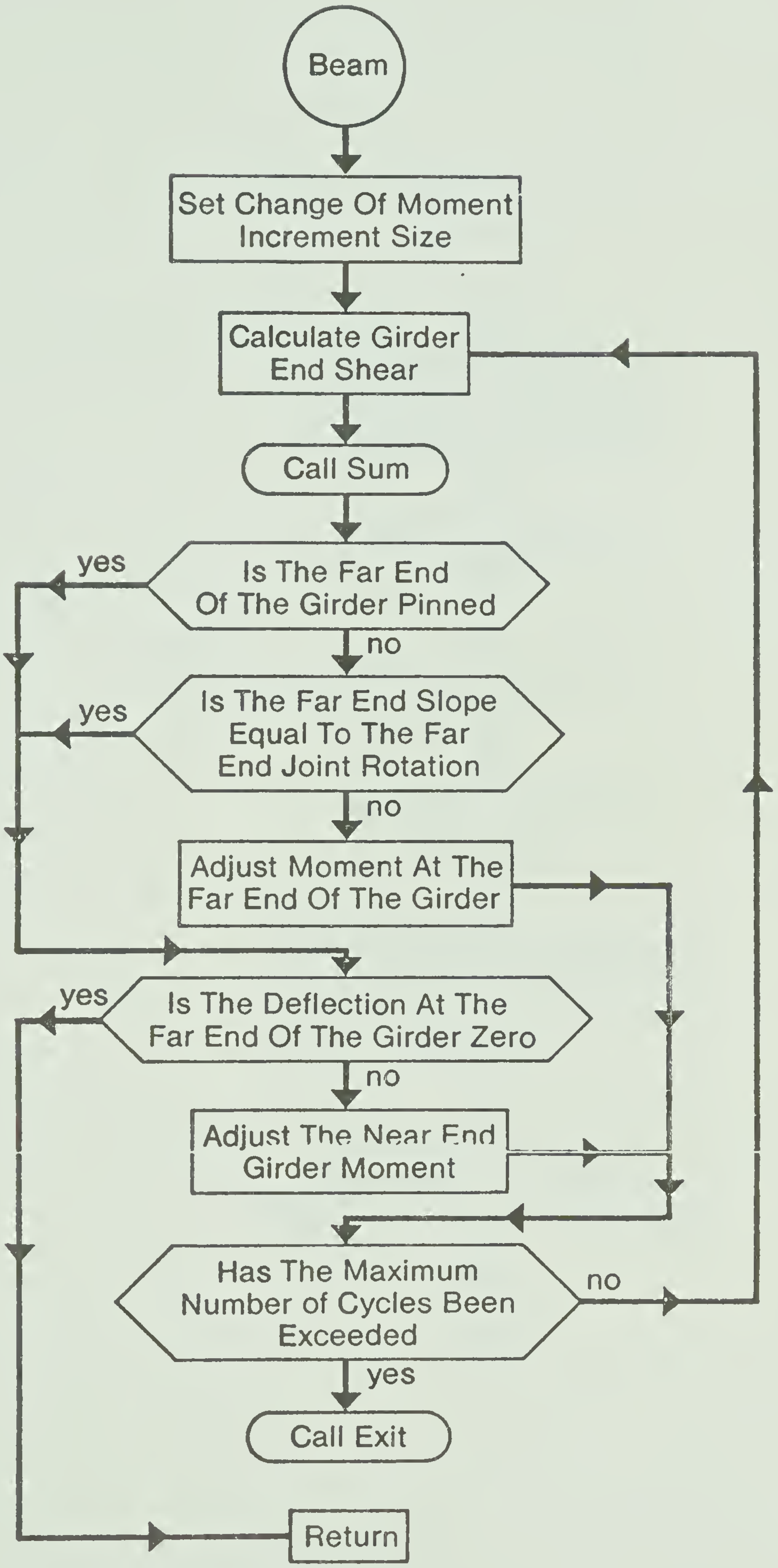


Figure B.5 Flow Diagram Subroutine Beam

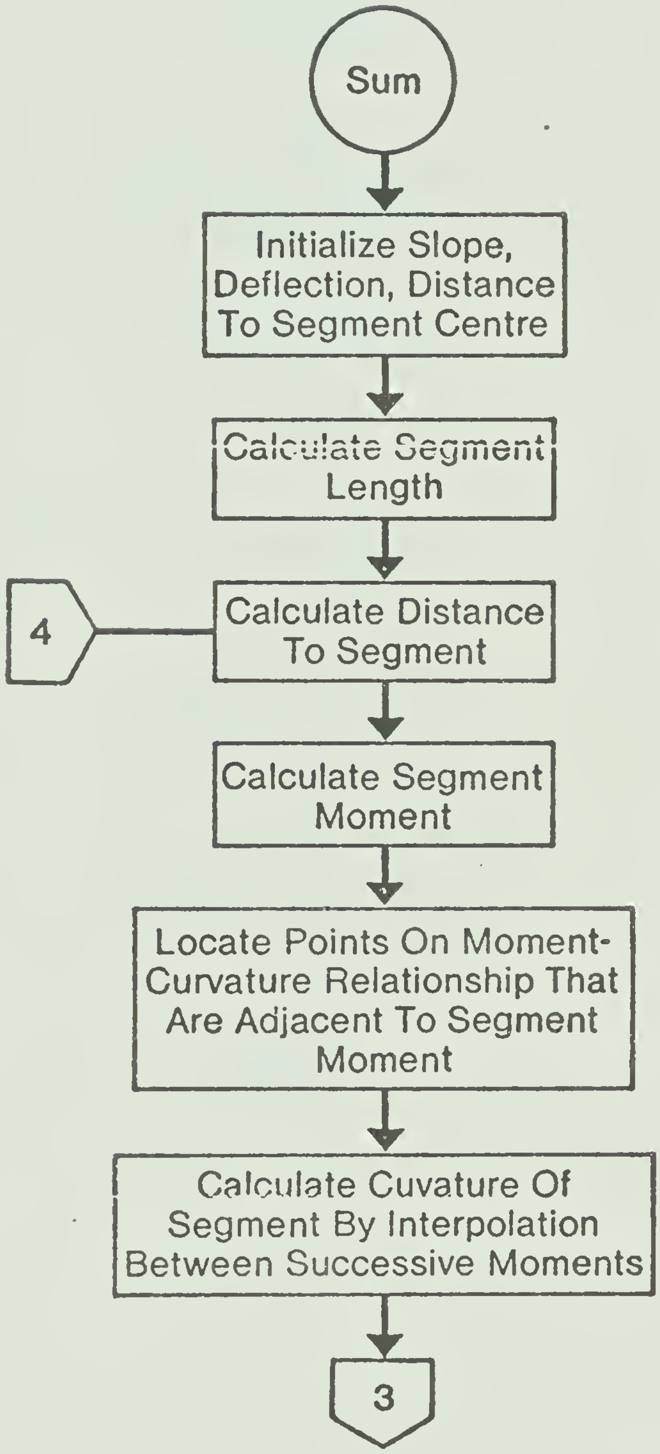


Figure B.6 Flow Diagram Subroutine Sum

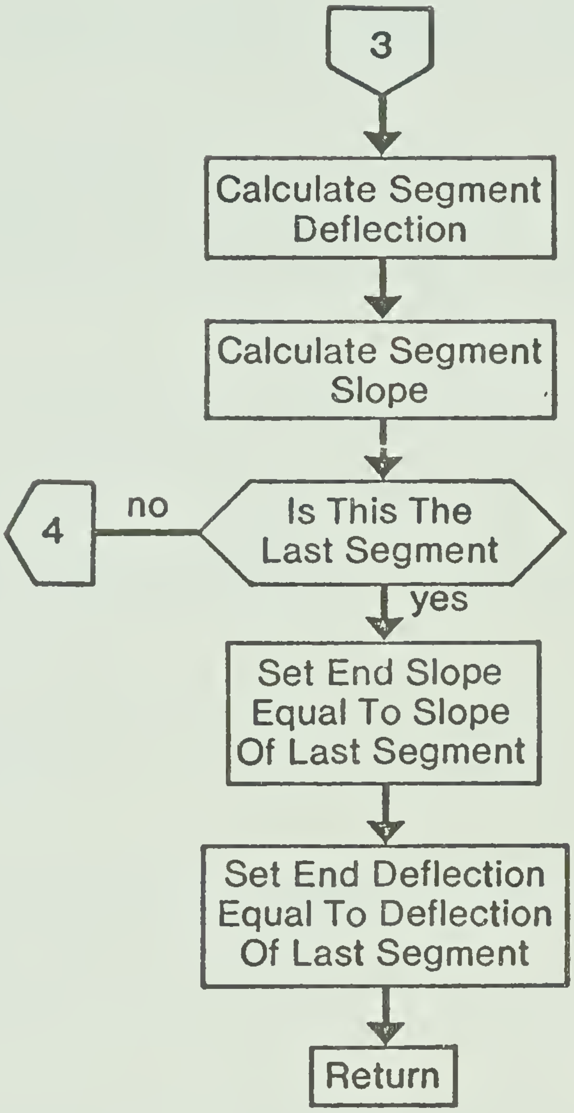


Figure B.6 Flow Diagram Subroutine Sum (continued)

B.3 INPUT DATA

The data for the program is input in the following order:

1. Problem identification label (2A6) column and girder properties identified (ie. elastic, inelastic, etc.) 2A6.

2. Girder properties (one card for each girder), section modulus, length, plastic modulus, load (4F10.0), if there is no beam on one side of the column set section modulus to zero.

3. Number of point in moment-curvature relationship (I2)

4. Girder curvature for moment-curvature data in increasing order starting from zero (8F10.0).

5. Girder moments for moment-curvature data in increasing order starting from zero (8F10.0).

6. Column data on one card, section modulus, length, plastic modulus, area, yield strength (5F10.0).

7. Number of point in the column moment-end rotation relationship (I2).

8. Corresponding column moments and end rotations in increasing order, one set for each card (2F10.0).

9. Column P/PY value (F10.0).

```
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION BM(20),BC(20),CR(20),CM(20)
   99 CALL READIN (BL,BQ,CL,CQ,NP,ROIMP,CR,CM,PM,NPC,BM,BC,THA,DTHA,NPB,
     1BI,BS,CI,CS,DI,DL,DZ,DA,FY,DP,DTHB,DTHC,INDEX)
      CALL CALC (BMB,BMC,PYB,PYC,PHIYB,PHIYC,BL,BQ,CL,CQ,PM,BI,BS,CI,CS,
     1DZ,DA,FY,DP,E,P,PY,BML,BMR,INDEX)
  800 DO 997 ID=1,NP
      TOL =DABS(THA*0.0025)
      THB=THA*DTHB
      THC=THA*DTHC
      IF (TOL.GT.0.0000005) TOL=0.0000005
      ABM=0.0
      IF (BI.LT.1.0) GO TO 100
      SIGN=1.0
      CALL BEAM (BL,BQ,ABM,TOL,PYB,PHIYB,ALPHAB,BMB,SIGN,THA,BM,BC,NPB,T
     1HB,BML,INDEX,ABV)
  100 ACM=0.0
      IF (CI.LT.1.0) GO TO 200
      SIGN=-1.0
      CALL BEAM (CL,CQ,ACM,TOL,PYC,PHIYC,ALPHAC,BMC,SIGN,THA,BM,BC,NPB,T
     1HC,BMR,INDEX,ACV)
  200 ADM=-(ABM+ACM+((ABV-ACV)*(DI*1.18/DZ)))/2.0
      PMC=CM(NPC)*PM
      ADM1=DABS(ADM)
      IF (ADM1.GT.PMC) GO TO 999
      CALL COLUMN (ADM,GA,CR,CM,PM,NPC)
      RO=THA-GA
      RO=RO+ROIMP
  600 RM=-ADM-(P*RO*DL)
      H=RM/PM
      BRM=DABS((2.*ADM)/PM)
      BMB=ABM/PYB
      BMC=ACM/PYC
      WRITE (6,70) H,RO,THA,BMB,BML,BMC,BMR,BRM
   70 FORMAT (/,F8.3,2F12.6,5F8.3)
      THA=THA+DTHA
  997 CONTINUE
      GO TO 99
  999 IF (N9.EQ.1) GO TO 991
      THA=THA-DTHA
      DTHA=DTHA/10.
      THA=THA+DTHA
      N9=1
      GO TO 800
  991 WRITE (6,998)
  998 FORMAT (///,'  COLUMN HAS FAILED ')
      STOP
      END
      SUBROUTINE READIN (BL,BQ,CL,CQ,NP,ROIMP,CR,CM,PM,NPC,BM,BC,THA,DTH
     1A,NPB,BI,BS,CI,CS,DI,DL,DZ,DA,FY,DP,DTHB,DTHC,INDEX)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION BM(20),BC(20),CR(20),CM(20)
   55 READ (5,67,END=9) FR,AME,COLUM,BEAM
   67 FORMAT (4A6)
      READ (5,1) BI,BL,BS,BQ
    1 FORMAT (4F10.0)
      READ (5,1) CI,CL,CS,CQ
```

```
      READ (5,2) NPB
    2 FORMAT (I2)
      READ (5,3) (BC(I),I=1,NPB)
      READ (5,3) (BM(I),I=1,NPB)
    3 FORMAT (8F10.0)
      READ (5,10) DI,DL,DZ,DA,FY
   10 FORMAT (5F10.0)
      READ (4,2) NPC
      DO 111 I=1,NPC
  111 READ (4,3) CM(I),CR(I)
      READ (4,4) PY
    4 FORMAT (F10.0)
      DP=PY*FY*DA
      WRITE (6,30) FR,AME,COLUM,BEAM
   30 FORMAT ('1 SOLUTION OF SUBASSEMLAGE ',2A6,/,'COLUMN IS ',A6,'BEAM
     1 IS ',A6,//)
      WRITE (6,5) PY
    5 FORMAT ('   P/PY=',F6.2,//)
      WRITE (6,40)
   40 FORMAT ('   BEAM PROPERTIES  ',//,'      I       L       S       Q
     1   ',/)
      WRITE (6,31) BI,BL,BS,BQ
      WRITE (6,31) CI,CL,CS,CQ
   31 FORMAT (4F8.1)
      WRITE (6,70)
   70 FORMAT (//,'       M/MP             PHI/PHI Y ',/)
      DO 71 I=1,NPB
   71 WRITE (6,72) BM(I),BC(I)
   72 FORMAT (2F14.4)
      WRITE (6,44)
   44 FORMAT (/ '   COLUMN PROPERTIES  ',//)
      WRITE (6,32) DI,DL,DZ,DA,FY,DP
   32 FORMAT ('      I       L       Z       A       FY      P  ',//,6
     1F8.1,//)
      WRITE (6,73)
   73 FORMAT (/,'      M/MPC             TH ',/)
      DO 74 I=1,NPC
   74 WRITE (6,75) CM(I),CR(I)
   75 FORMAT (2F15.6)
      READ (5,66) THA,DTHA,ROIMP,DTHB,DTHC,NP
   66 FORMAT (5F10.0,I5)
      WRITE (6,482) DTHB,DTHC
  482 FORMAT (/,'   THA=',F7.3,' THB    THC=',F7.3,' THB ',/)
      WRITE (6,483) ROIMP
  483 FORMAT ('   INITIAL IMPERFECTION  RO =',F10.6,/)
      INDEX=1
      IF (DTHB.LT.0.001) INDEX=2
      WRITE (6,46)
   46 FORMAT('1     H           RO         THETA    MBA     MAB    MBC
     1  MCB       MR/MPC ')
      GO TO 8
    9 CALL EXIT
    8 RETURN
      END
      SUBROUTINE CALC (BMB,BMC,PYB,PYC,PHIYB,PHIYC,BL,BQ,CL,CQ,PM,BI,BS,
     1CI,CS,DZ,DA,FY,DP,E,P,PY,BML,BMR,INDEX)
      IMPLICIT REAL*8(A-H,O-Z)
      E=29000.
      BQ=BQ/12.
      CQ=CQ/12.
```

```
      PYB=PY*BS
      PYC=PY*CS
      IF (INDEX.EQ.1) GO TO 100
      P=DP
      FMB=BQ*BL**2/8.
      FMC=CQ*CL**2/8.
      BML=0.0
      BMR=0.0
      GO TO 200
  100 P=DP
      FMB=BQ*BL**2/12.
      FMC=CQ*CL**2/12.
      BML=-FMB/PYB
      BMR=FMC/PYC
  200 BMB=FMB/PYB
      BMC=-FMC/PYC
      PHIYB=PYB/E/BI
      PHIYC=PYC/E/CI
      PY=PY*DA
      PM=1.18*DZ*PY*(1.-P/PY)
      RETURN
      END
      SUBROUTINE COLUMN (ADM,GA,CR,CM,PM,NPC)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION BM(20),BC(20),CR(20),CM(20)
      ADMY=ADM/PM
      SIGN=1.
      IF (ADMY.LT.0.0) SIGN=-1.
      ADMY=DABS(ADMY)
      DO 29 J=1,NPC
      IF (ADMY.LT.CM(J)) GO TO 28
   29 CONTINUE
   28 SLOPE=(CM(J)-CM(J-1))/(CR(J)-CR(J-1))
      GA=(((ADMY-CM(J-1))/SLOPE)+(CR(J-1)))*SIGN
      RETURN
      END
      SUBROUTINE SUM (XL,XQ,AXM,AXV,PYX,PHIYX,ALPHAX,SIGN,BM,BC,THA,NPB,
     1TH)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION BM(20),BC(20),CR(20),CM(20)
      TH=THA
      X=0.0
      V=0.0
      DELTA=XL/50.
      DO 100 N=1,50
      RN=N
      SIGNB=1.
      X=RN*DELTA-(DELTA/2.)
      BMD=AXV*X-XQ*X**2/2.-(SIGN*AXM)
      IF(BMD.GT.0.0) SIGNB=-1.
      BMD=DABS(BMD/PYX)
      DO 9 J=1,NPB
      IF (BMD.LT.BM(J)) GO TO 8
    9 CONTINUE
    8 SLOPE=(BM(J)-BM(J-1))/(BC(J)-BC(J-1))
      PHI=(((BMD-BM(J-1))/SLOPE)+BC(J-1))*PHIYX*SIGNB*SIGN
      V=V-PHI*DELTA**2/2.+TH*DELTA*SIGN
      TH=TH-PHI*DELTA
      XI=RN*DELTA
      BMD=BMD*(-SIGNB)
```

```
  2 FORMAT(F6.1,4F15.8)
100 CONTINUE
    ALPHAX=V/XL
666 FORMAT (4F14.9)
130 RETURN
    END
    SUBROUTINE BEAM (XL,XQ,AXM,TOL,PYX,PHIYX,ALPHAX,BMX,SIGN,THA,BM,BC
   1,NPB,THX,BMF,INDEX,AXV)
    IMPLICIT REAL*8(A-H,O-Z)
    DIMENSION BM(20),BC(20),CR(20),CM(20)
    DM=0.010
    IF (BMX.EQ.0.0) BMX=10.*THA
    DO 800 NX=1,100
    JX=NX/2
    RX=NX
    SX=R/2.
    TX=SX-NX
    DM=0.010
    DO 200 NT=1,200
    DM2=0.01
    DO 500 NQ=1,200
    AXM=BMX*PYX
    XAM=BMF*PYX
    AXV=((BMX*SIGN*PYX)+(BMF*SIGN*PYX)+((XQ*(XL**2))/2))/XL
    CALL SUM (XL,XQ,AXM,AXV,PYX,PHIYX,ALPHAX,SIGN,BM,BC,THA,NPB,TH)
    IF (INDEX.EQ.2) GO TO 400
    TOL2=TOL
    DTH1=(THX-TH)
    DTH2=DABS(DTH1)
    IF (TX.GT.0.0) GO TO 400
    IF (DTH2.LT.TOL2) GO TO 400
    IF (NQ.EQ.1) GO TO 600
    DIFF2=DTH3/DTH1
    IF (DIFF2.GT.0.0) GO TO 600
    BMF=BMF-(DM2*SIGNC)
    DM2=DM2/10.
    GO TO 500
600 DTH3=DTH1
    SIGNC=1.0
    IF (DTH1.LT.0.0) SIGNC=-1.0
    BMF=BMF+(DM2*SIGNC)
500 CONTINUE
    WRITE (6,778)
    CALL EXIT
400 ALX=DABS(ALPHAX)
    IF (ALX.LT.TOL) GO TO 888
    IF (NT.EQ.1) GO TO 100
    DIFF=ALPHAX/ALX1
    IF (DIFF.GT.0.0) GO TO 100
    BMX=BMX-(DM*SIGND*SIGN)
    DM=DM/10.
    GO TO 200
100 ALX1=ALPHAX
    SIGND=1.0
    IF (ALPHAX.LT.0.0) SIGND=-1.0
    BMX=BMX+(DM*SIGN*SIGND)
200 CONTINUE
    WRITE (6,779)
777 FORMAT (///'   NO CONVERGENCE ')
779 FORMAT (///'   NO CONVERGENCE 2 ')
```

```
      778 FORMAT (///'    NO CONVERGENCE 1')
          CALL EXIT
      888 IF (INDEX.EQ.2) GO TO 999
          IF (ALX.LT.TOL.AND.DTH2.LT.TOL2) GO TO 999
      800 CONTINUE
          WRITE (6,777)
      999 AXM=BMX*PYX
          RETURN
          END
FILE
```